Audit - if necessary, instead
Plot groups: publication record.

Econ 609 (Aew) 09/10/2008

Value \( f^* \) - having differentiability

\( \Rightarrow \) Any way is to get concavity & then rule out bounds by looking for value \( f^* \) below by \( v \).

\[ V^* \text{ vs. } V^+ \]

\( V \) - at least as good as doing it cheaply.

Bellman Equation:

\[ V_t(W) = \max_{c \in [0, W_t]} \{ \mathbb{E}[U(c_t) + \beta \mathbb{E}\{V_{t+1}(b_t(W_{t+1}) + \nu_{t+1})]\} \]

How to prove properties of \( V^2 \) (Backward recursion)

0. \( V_{t+1} \) EP (\( V \) has some property \( P \) ) \( V_{t+1}(W_{t+1}) = 0 \)

1. Recursion

a) Show \( V_{t+1} \) EP \( = F_t(K_t, X_t) \in \mathbb{Q} \)

b) Show \( F_t \in \mathbb{Q} = V^1 \) EP (preservation under reinforcement)

\[ V^*(K_t) = \max_{X_t} F_t(K_t, X_t) \quad V^*(W_t) = \max_c F^t(W_t, c) \]

Heed \# to behave as under reinforcement and expectation to extend to property in \( F_t(\cdot, \cdot) \).

Subject selection

*Lifetime utility* \( f^u \): \( V_0 = \mathbb{E}_{\pi}\sum_{t=0}^{T} \beta^t U(c_t) \)

*Planning the agent:

\[ \max_{\pi} \mathbb{E}[\sum_{t=0}^{T} \beta^t U(c_t) + \beta \mathbb{E}\{V_{t+1}(b_t(W_{t+1}) + \nu_{t+1})]\}

\[ \Rightarrow \]

\[ \mathbb{E}[V_{t+1}(b_t(W_{t+1}) + \nu_{t+1})] \]

\[ \mathbb{E} \{ U(c_t) \} \]
$$K_{th} = P(K, X, Random \ V_X) \quad \text{conclusion of state model}$$

In an example: \( K_{th} = R(w - c) + \gamma_{th} \)

**Proof:**

\( V(w) > 0 \) - our way out of the loop.

\( V(w) = 0 \)

\( V_{th} > 0 \) (if \( \gamma_{th} \), \( \gamma_{th} \), \( \beta \))

(**Note:** parameter can be argument)

Case: \( V_{th} \) - always hold

\( V_{th} = \sum_{i=0}^{\infty} \gamma_i \) - structure of the problem

\( V(w + s) - V(w) \geq 0 \quad \forall s > 0 \)

We need to subscript

\( V(w + s) - V(w) \geq 0 \) \( (w \text{ to a dummy } w_0) \)

\( = \max_{c \in \mathcal{C}} \{ u^*(c) + \beta \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c) + \gamma_{th}) \} \)

\( c^{*} \quad \text{(let } c^{*} \text{ be the maximum)} \)

\( = \max_{c \in \mathcal{C}} \{ u^*(c) + \beta \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c) + \gamma_{th}) \} \quad \text{organ} \)

\( \geq u^*(c^{*}) + \beta \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c^{*}) + \gamma_{th}) \quad \text{by definition} \)

\( > u^*(c^{*}) + \beta \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c^{*}) + \gamma_{th}) \quad \text{by maximal value of } \gamma_{th} \)

\( = \beta \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c^{*}) + \gamma_{th}) \quad \text{by definition} \)

\( \geq 0 \quad \text{by definition} \)

\( \lambda \mathbb{E} V^{th} = \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c^{*}) + \gamma_{th}) \)

We know \( \lambda \mathbb{E} V^{th} \text{ true for } \lambda \)

\( \text{We were doing recursion!} \)

\( \geq \mathbb{E} V^{th} = \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c^{*}) + \gamma_{th}) \)

To get strictness - we need previous \( c^{*} \)

**Note:**

**Have no property of utility** that matters

- Could you prove in the same way that value is decreasing?

If closer to base by one:

\( \leq u^*(c^{*}) + \beta \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c^{*}) + \gamma_{th}) \)

\( = u^*(c^{*}) + \beta \mathbb{E} V^{th}(\hat{R}_{th}(w - c^{*}) + \gamma_{th}) \)

\( \text{But now } \gamma_{th} \text{ is not in the interest!} \)

See Theorem \( \lambda \mathbb{E} V^{th} = \mathbb{E} V^{th}(\hat{R}_{th}(w + s - c^{*}) + \gamma_{th}) \)

- Since \( \gamma_{th} \) not in the set
\[ U^t(W + s; T) - U^t(W; T) \geq 0 \quad \text{(weaker t and finite T)} \]

Suppose

\[ \lim_{T \to \infty} U^t(W, T) = U^t(W, \infty) \quad \text{(limit exists)} \]

Then

\[ U^t(W + s, \infty) - U^t(W, \infty) = \lim_{T \to \infty} [U^t(W + s, T) - U^t(W, T)] \geq 0 \]

Note: did not need any assumption on utility \( f^t \).

\[ U^t(K_t) = \max_{k_t, x} f^t(k_t, x) \]

To prove continuity:

\[ V^t(K_t) \]

More recent lie - more advanced way of thinking.

Claim: \( V^t(\theta_k, (1-\theta)k_2) - \theta V(k_1) - (1-\theta) V(k_2) \geq 0 \).

Recent lie - weighted average of \( V^t \) at schedule:

\[ \max_{k_t, x} f^t(\theta_k, (1-\theta)k_2, x) - \theta \max_{k_t, x} f^t(k_1, x) - (1-\theta) \max_{k_t, x} f^t(k_2, x) \]

\[ \geq f^t(\theta_k, (1-\theta)k_2, x) - \theta f^t(k_1, x) - (1-\theta) f^t(k_2, x) \]

\[ \text{if } x \in X(\theta_k, (1-\theta)k_2) \]

\[ \text{if } x \in X(\theta_k, k_1) \]

What if we plug \( f^t \) do we need to worry about this \( \geq 0 \)?
\[ \frac{T}{\text{sec}} \cdot \frac{\text{sec}}{\text{m}} = \text{m} \]

Due in one week

**Present Week**

**Due in one week**

**Problem:**

- **Objective Function:** Minimize \( f(x) \)
- **Constraints:** \( g(x) \geq 0 \)

**Solution Steps:**

1. **Initial Setup:**
   - **First-order conditions:**
     - \( f(x) = c(x) - c'(x) \cdot k \)
     - \( \frac{\partial f}{\partial x} = c'(x) \cdot k + c''(x) \cdot k^2 = 0 \)
   - **Second-order conditions:**
     - \( c''(x) \cdot k^2 < 0 \)
     - \( f''(x) = c''(x) \cdot k^2 > 0 \)

2. **Feasibility:**
   - **Feasible region:** \( V(x) \geq 0 \)
   - **Optimality:** \( V(x) = 0 \)

3. **Iterative Process:**
   - **Step 1:**
     - \( V(x) = \max F(K, x) = (1 - \theta) F(K, x) - \theta F(K, x) \)
     - \( \theta \in [0, 1] \)
     - \( x^* = \arg \max F(K, x) \)
   - **Step 2:**
     - \( x^* = \frac{\partial F}{\partial x} \)
     - **Convergence:**
       - If \( x \) is feasible, then \( x^* \) is optimal.

**Conclusion:**

- **Feasible region:** \( V(x) \geq 0 \)
- **Optimal solution:** \( x^* \) is optimal.

**Additional Notes:**

- **Homework:**
  - **Problem 1:**
    - In a firm's problem, find and post reasonable conditions under which the value \( f'' \) in objective function reduces to scale.
  - **Problem 2:**
    - Modify \( f'' \) to be zero and find the critical points.

**Notes:**

- **Symmetry:**
  - **Homogeneous:**
    - In the case of homogeneous functions, the value function reduces to scale.
  - **CRS:**
    - Constant Returns to Scale (CRS) symmetry.

**Date:** 14/01/2007
Suppose \( k \) can really take an integer value \( \mathbb{Z} \) or \( X \), not count \( m \) in \( \mathbb{R}^2 \).

Still works whenever \( \mathbb{O}_k + (1 - \alpha) P_k \) is a feasible choice of \( k \).

Just need the variance of \( X \) to be in the set.

What if \( X \) discrete?

\[
F(k, X) \rightarrow \text{Soldier of Fortune} \quad F(k, 0)
\]

- It less gamble to get high utility
- Lower certainty can make \( m \) risk bearing!

Take \( k \) - plot concave (even though \( f(k) \) convex)

- Discrete choice can lead to non-concave variance (value \( f(k) \))
- Proving joint convexity of \( F(\cdot, \cdot) \) in \( k, X \):

\[
V^T(W) = \max \mathcal{U}(c) + \beta \mathbb{E}_c V^{T_1}(F_{11}(W - c) + 7m_1).
\]

- \( V^T \) concave - taking concave at base - concave
- Proving \( \mathbb{E}_c \) concave in \( c \):

\[
V^T(W) = \max \mathcal{U}(c) + \beta \mathbb{E}_c V^{T_1}(F_{11}(W - c) + 7m_1).
\]

[Writing it recursively assumes no time inconsistency]

\[
V^T(k, X) = \max \mathcal{U}(c) + \beta \mathbb{E}_c \left[ \mathbb{E}[V^T(X, k, \bar{c}) + 7m_1] \right].
\]

- \( \hat{c} \) is known (state variable in future) - stochastic discount factor

- With policy \( c(t), c(\cdot) \)
\[ \pi(k, n) = \max_{n} \pi(k, n) \] 

\[ \pi(k) = \max_{n} \pi(k, n) \] 

Optimization sub-problem needs the right and at each period.

What kind of facts about \( \pi(k) \) will deliver these kinds of facts about \( \pi(k, n) \)?

3. Think about what economic environment you need to get \( \pi(k, n) \) of the slope you need.

\[ \pi^*(w, \beta) = \max_{c} \pi(c) + \beta \mathbb{E} V^{**}(k_0, (w-c) + \gamma H, \beta) \]

\[ \text{Focus: } \frac{\partial \pi^*(w, \beta)}{\partial w} \]

\[ V^*(w, \beta) = \beta \mathbb{E} \frac{\partial V^*(w-c) + \gamma H, \beta}{\partial w} = 0 \]

\[ V^*(w, \beta) = \beta \mathbb{E} \frac{\partial V^*(w-c) + \gamma H, \beta}{\partial w} \]

\[ V^*(w, \beta) = \beta \mathbb{E} \frac{\partial V^*(w-c) + \gamma H, \beta}{\partial w} \]

\[ V^*(w, \beta) = \beta \mathbb{E} \frac{\partial V^*(w-c) + \gamma H, \beta}{\partial w} \]

Slope of upper envelope: demand constraint curve

For extreme choice of \( c \):

\[ \pi^*(w, \beta) \]

What property at which \( \beta \) will make \( \beta \rightarrow c^* \)?

\[ \pi_{\beta}(w, \beta) \geq 0 \] (monotonicity of \( V^* \) in \( w \) w.r.t. \( \beta \))

\[ \beta \rightarrow \pi^*(w, \beta) \geq c^* \cdot \pi^*(w, \beta) \]

\[ \text{Def. } (\text{of supermodularity}) \]

\[ K \text{ is a big vector} \]
\[ V(k_1, V k_2) + V(k_1 \land k_2) = V(k_1) - W(k_2) > 0 \]

- \( k_1 \lor k_2 \) - conv. win max

- \( k_1 \land k_2 \) - conv. win min

\( \land = \cap \) - intersection - small

Joint supermodularity for a component = joint supermodularity for all parts of components

\( \mathbf{K}_1 = (K_1, I_1) \) - limits of components

\( \mathbf{K}_2 = (K_2, I_2) \)
Econ 609 (lec) 16/01/2008

* Define:\[ S = W - C - \text{exogenous part of } W - \text{net profits}\]

\[ K_0 = (\frac{w_0}{\beta}) = (\frac{w_0}{\beta}) \]

\[ K_1 = (w_1, \beta) \]

\[ K_2 = (w_2, \beta) \]

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\[
V^*(W, \beta) = \max_S u(W - S) + \beta E_t V_t^*(R_t(S + \gamma_t, \beta))
\]

\[
F_u = u'(W - S) + \beta E_t V_t^*(\omega_t)
\]
\[
F_R = E_t R_t V_t^*(R_t(S + \gamma_t, \beta)) + \beta E_t V_t^*(R_t(S + \gamma_t, \beta))
\]
\[
F_S = -u'(W - S) + \beta E_t R_t V_t^*(R_t(S + \gamma_t, \beta))
\]

\[
F_p = 0 \quad \text{gap new clue}
\]
\[
F_u = -u'(W - S) > 0 \quad \text{gap new clue}
\]

\[
F_p = F_R - E_t R_t V_t^*(R_t(S + \gamma_t, \beta)) + \beta E_t R_t V_t^*(R_t(S + \gamma_t, \beta)) > 0
\]

Even though, let stochastic process go on, one patient will have error.

At \( T \):
\[
V^*(W, \beta) = \max_S u(W - S) = V_{\text{gap}} = 0
\]

\[
V_{k+1} = V_k + \alpha
\]

\[
V_k = 0
\]

We know that \( S \) is not known yet.

\[
\text{Back to firm's problem:}
\]
\[
V^*(X, K + \alpha) = \max_{X, K + \alpha} \left[ \frac{\mu(X, K + \alpha)}{P(Y_t, Z_t)} + E_t D_t^*(\beta, \gamma_t) V_t^*(X, K + \alpha, \beta, \gamma_t) \right]
\]

\[
\text{Foc:}
\]
\[
P(Y_t, Z_t) C_X(X, K, \beta_t) + E_t D_t^*(\beta, \gamma_t) V_t^*(X, K, \beta, \gamma_t) = 0
\]

\[
\text{Foc:}
\]
\[
P(Y_t, Z_t) C_X(X, K, \beta_t) = E_t D_t^*(\beta, \gamma_t) V_t^*(X, K, \beta, \gamma_t)
\]
If $V_x > 0$

$\Rightarrow C_x < 0$

Merotene Camp: Statics

\[ F_x \]

\[ x \]

No sensitivity: $m \neq 0$

\[ F_x \]

\[ x \]

Likely to jump -- war will jump around

\[ F_x \]

\[ x \]

Kilgour + Sanan

$F_{xx} > 0$ (like superconductivity)

* Strong set order

Don't need sensitivity to push man to the right

$\alpha$ like demand
\[
V^0(x, k, \beta, \gamma; \zeta) = \max_x \left\{ T(k, x, \zeta) - P(x, \beta) C(x, k, \beta) + \right. \\
\left. \min_{\beta, \gamma} \max \left\{ T(k, x, \beta, \gamma) \right\} \right\}
\]

\[
\text{Suppose } F_{xx} \geq 0, \text{ then } x^* = F_x(0) = F_{xx}(0) - \frac{1}{2} F_{xxx} \left( x^* \right)^2
\]

Next move: Optimal value to the right.

2nd order: \( F_{xx} > 0 \) - flat out & concave.

Break here! Can evaluate at all coordinate points.

Candidate 1 is... move to the right, is... invalid.

If it disappears, clearly need to... move to the right.

Take next candidate point.

Candidates shift to the right. Don't need certainty.

Concavity is needed for second order.

of optimal value of \( x \).

Note: 1st could also be valid.

Need to prove something about \( V_{xx} \) to get \( F_{xx} \). (Which we want)

Last point: \( V^0(x, k, \beta, \gamma) = \max \left\{ T(k, x, \beta, \gamma) - P(x, \beta) C(x, k, \beta) \right\} \). Can we put together?

\[ V_{xx} \leq 0 \Rightarrow V_{xx} = 0 \Rightarrow \text{Can we use this?} \]

No reason to care about \( x \). (No period in \( T^{x1} \) - set \( x \) to constant)

Might as well assume \( \Pi_{xx} 20 \). (Ignore what about this)

Need to worry about supercontouring \( k, x, \mu \) (larger set).

\[
\text{If } V_{xx} \geq 0 \text{ then } F_{xx} \geq 0. \quad \text{If } F_{xx} \geq 0 \quad \text{then } V_{xx} \geq 0
\]

If \( F_{xx} \geq 0 \), then \( F_{xx} \geq 0 \).

if \( F_{xx} \geq 0 \), then \( V_{xx} \geq 0 \).

if \( F_{xx} \geq 0 \), then \( F_{xx} \geq 0 \).

\[
F_{xx} = F_{xx}(x, k, \beta, \gamma) \geq 0, \quad \text{and } F_{xx} \geq 0
\]
What does this mean?

If \( \forall \mathbf{X} \geq 0 \) and \( C_{\mathbf{X}} \geq 0 \) for \( \forall \mathbf{X} \geq 0 \) and \( F_{\mathbf{X}} \geq 0 \),

\[ \Rightarrow \mathbf{X} \rightarrow \mathbf{X}. \]

No adjustment on \( \mathbf{X} \),

- \( \mathbf{X} = [f(t) \mathbf{X} + \mathbf{I}] \)
  - \( \mathbf{X} = e^{\mathbf{X} \mathbf{I} + \mathbf{I}} \) (continuous time)
  - \( \mathbf{I} = \mathbf{X} - (1-\mathbf{I}) \mathbf{I} \)
  - \( \mathbf{I} = e^{\mathbf{X} \mathbf{I} + \mathbf{I}} \) (continuous time)


Finally, \( F_{\mathbf{X}} \geq 0 \implies (C_{\mathbf{X}} \mathbf{X}) = \mathbf{I} - \mathbf{X} - (1-\mathbf{I}) \mathbf{I} \)

\[ K \mathbf{J} = \left( \frac{\mathbf{I}}{\mathbf{J}} \right) \quad J \gg 0 \quad (\mathbf{J} \text{-the largest}) \]

\[ C_{\mathbf{X}} \mathbf{X} = K \mathbf{J} \left( \mathbf{X} + (1-\mathbf{I}) \mathbf{I} \right) - K \mathbf{J} \left( \frac{\mathbf{I}}{\mathbf{J}} - (1-\mathbf{I}) \mathbf{I} \right) \]

\[ C_{\mathbf{X}} \mathbf{X} = K \mathbf{J} \left( \frac{\mathbf{I}}{\mathbf{J}} - (1-\mathbf{I}) \mathbf{I} \right) \]

\[ \mathbf{X} \rightarrow \mathbf{X} \]

\[ C_{\mathbf{X}} \mathbf{X} = K \mathbf{J} \left( \frac{\mathbf{I}}{\mathbf{J}} - (1-\mathbf{I}) \mathbf{I} \right) \]

\[ C_{\mathbf{X}} \mathbf{X} = - \frac{\mathbf{X}}{\mathbf{J}} \mathbf{J} \left( \frac{\mathbf{I}}{\mathbf{J}} - (1-\mathbf{I}) \mathbf{I} \right) \]

\[ \mathbf{X} \rightarrow \mathbf{X} \]

\[ V_{\mathbf{X}} \geq 0 \Rightarrow F_{\mathbf{X}} \geq 0 \]

\[ F_{\mathbf{X}} \geq 0 \Rightarrow F_{\mathbf{X}} \geq 0 \]

\[ \mathbf{X} \rightarrow \mathbf{X} \]

\[ F_{\mathbf{X}} \geq 0 \Rightarrow F_{\mathbf{X}} \geq 0 \]

\[ \mathbf{X} \rightarrow \mathbf{X} \]

\[ F_{\mathbf{X}} \geq 0 \rightarrow \mathbf{X} \geq 0 \]

\[ \mathbf{X} \rightarrow \mathbf{X} \]
\[ V_t( k_t, x_t, \beta, \gamma, z_t) = \max \left( T(k_t, x_t, z_t) - (y(x_t, C(x_t, k_t, z_t)) + \delta_t V_{t+1}(k_{t+1}, x_{t+1}) \right) \]

To show: \[ F( x_t) \leq 0 \] (negative price function instead)

Assume \[ F( x_t) < 0 \]

\[ F( x_t) = -P_t C_t + \delta_t D_t(\beta, \gamma, z_t) V_{t+1}(x_{t+1}, \beta, \gamma, z_{t+1}) \]

"Hold" \[ V_{x_t} \geq 0 \], i.e. \[ V \text{ subordinated in } k_t, x_t \]

Assume \[ C_t \geq 0 \] (costs remaining coming to investment tomorrow)

Together with the initial condition \[ F(\beta, \gamma, z_t) \text{ supermodular in } x_t \text{ to } (1-\delta_t)k_t \]

\[ F( x_t) = -P_t C_t + \delta_t D_t(\beta, \gamma, z_t) V_{t+1}(x_{t+1}, \beta, \gamma, z_{t+1}) \text{ for } x_{t+1} \geq 0 \]

\[ F( x_t) = 0 \text{ (need } C_t \leq 0) \]

\[ F( x_t) = \text{ building of } x_t \text{ and } \delta_t 

F( x_t) \leq 0 \] (as long as \[ V_{x_t} \geq 0 \] \[ \implies \text{ can get } C_t \text{ by reduction})

\[ \text{F}( x_t) = -P_t C_t + \delta_t D_t(\beta, \gamma, z_t) V_{t+1}(x_{t+1}, \beta, \gamma, z_{t+1}) \leq 0 \]

\[ C_t \geq 0 \] \[ \text{need } V_{x_t} \leq 0 \]

What about a temporary tax credit? \[ V_{t+1}(x_{t+1}, \beta, \gamma, z_{t+1}) \]

\[ F( x_t) = -P_t C_t + \delta_t D_t(\beta, \gamma, z_t) V_{t+1}(x_{t+1}, \beta, \gamma, z_{t+1}) \leq 0 \]

\[ C_t \geq 0 \] \[ \text{need } V_{x_t} \leq 0 \]

But the lower bound also has an impact you might get

Characteristics of a steady state, and the best that you can achieve down the road,

\[ \text{given the problem a lot of structure} \]

For... doing things in our generation - replicates of steady state etc.

\[\text{set } \delta P_t \leq 0 \]

\[ F( x_t) = \delta_t D_t(\beta, \gamma, z_t) V_{t+1}(x_{t+1}, \beta, \gamma, z_{t+1}) \]

\[ \text{lower } x_t \]

\[ F( x_t) = \delta_t D_t(\beta, \gamma, z_t) V_{t+1}(x_{t+1}, \beta, \gamma, z_{t+1}) \]

\[ \text{want } F( x_t) \leq 0 \]

\[ \text{if } F( x_t) \leq 0 \]

1. \[ x_t \geq 0 \]

\[ V(x_t) = \max F(k, x_t) \]

\[ V(k_t, x_t) = \max F(k_t, x_t) \]

\[ \geq F(k_t, x_t) \]

\[ x_t \in \text{ feasible set } \]

\[ \geq F(k_t, x_t) \]

\[ \geq 0 \text{ for } F(k_t, x_t) \]

\[ \text{PTO} \]
Could have used Envelope Theorem.

If you push up objective function

Then the marginal line goes up.

Time cannot be differentiated.

Case 2: \( V_e = 0 \)

Then \( F_e = 0 \).

So \( F_e = 0 \).

Check more structure.

Define \( g_e^{(k, x, y, z)} = \frac{V_e^{(k, x, y, z)}}{p^e(y)} \).

Denote by \( \Delta^{(k, x, y, z)} \).

Then \( g_e^{(k, x, y, z)} = \max \frac{\Pi^{(k, x, y, z)}}{p^e(y)} = \frac{C(x, y, z)}{p^e(y)} \).

\( \Delta^{(k, x, y, z)} = \frac{D^{(k, x, y, z)}(c, p, y) - P^{e(y)}}{p^e(y)} \).

\( \Delta^{(k, x, y, z)} = b(t(x, y, z)) \).
Example: Bellman's Equation for a diffusion process (x, t) 

\[ \begin{align*} 
V'(x) &= V_t(x) = \max \left\{ U(x_k, x) + V(x_k) A(x_k, x) + V(x_k) B(x_k, x) \right\} \\
\text{maximize} & \quad \frac{dV}{dx} \\
V(x) &= \max \left\{ U(x_k, x) + e^{-\frac{x^2}{2}} \right\} \\
\text{subject to} & \quad V(x) = 1, \quad \text{for} \quad x \to \infty \\
\text{constraint:} & \quad \text{Hidden Markov Model} \\
\text{initial condition:} & \quad V(x) = 1, \quad \text{for} \quad x \to \infty \\
\text{boundary condition:} & \quad V(x) = 1, \quad \text{for} \quad x \to \infty \\
\text{solve by:} & \quad \text{Dynamic Programming} \\
\text{for} \quad x \to \infty & \quad \text{Bellman's Equation} \\
\text{Sequence of 2-point risk can span any diffusion} \\
\text{e.g., to span a diffusion by} & \quad \text{Black-Scholes} \\
\text{span diffusion} & \quad \text{e.g., Thomas (1994) and Pagliero} \\
\text{As long as you're spanning the space, this up-and-out technique will work} \\
\text{Simpson's own ch?} & \quad \text{Partly complete methods whenever there is a tradable} \\
\text{once asset, there is also a full set of option-on that asset (e.g., equivalent)).} \\
\text{price things} & \quad \text{in crashes twice my 2-point risk and then take the limit} \\
\text{Hidden problem: Individual Sourcing for return on trade} & \quad \text{means very cheap to hold} \\
\text{max} & \quad E \left[ \int_0^\infty e^{-r t} dt \right] \\
\text{constraint:} & \quad \text{expected utility} \\
K_0 \text{ fixed - and - expected wealth } & \quad dK = (r + x_K - c) dt + \sigma dW \\
\text{For example possible if you have} & \quad dX = \sqrt{t} \sigma \, dW, \quad \sigma \text{- std dev per sqrt time unit} \\
\text{mu - risk to pull down, } c \text{- easy to pull down, } \sigma \text{ - empirically} \\
\end{align*} \]
\[ f(x^2) \]

One leg is scaled and one-valued.

\[ y = \frac{1}{2} x \cdot x \]

lead to transformation to non-differentiable:

\[ \frac{\partial^2}{\partial x^2} \]

\[ k \rightarrow k \alpha \]

\[ c \rightarrow c \alpha \]

\[ \alpha \rightarrow \alpha \theta \]

\[ y \rightarrow \theta^{1-y} \]

will work nicely through \( \Psi(x) \).

\[ \Pi \rightarrow \frac{\alpha}{\alpha - 1} \]

Does it respect the constraints? yes.

\[ \alpha \in [0, 1] \Rightarrow \theta \in [0, \alpha k] \]

\( \theta > 1 \)

\( \theta \) will work when in list,

\[ \alpha \in [0, 1] \Rightarrow \theta \in [0, \alpha k] \]

\[ \text{before to be sure if it holds or if we need some other} \]

\[ dk = (rk + c\mu - c) dt + c \sigma dz \]

\[ \delta \Psi(k) = (\theta(k) + \epsilon) \sigma + o(\epsilon) \sigma \]

\[ \Psi(k) = k(1-k) \]

Then \( \Psi(k) = \theta^{1-y} \Psi(k) \) in the domain of \( k \) we maintain scale

To see the slope of \( \Psi(k) \), just change \( \alpha \rightarrow \frac{1}{\alpha} \)

\[ \Psi'(k) = \frac{1}{\alpha} \Psi(k) \]

\[ \Psi(k) = k(1-k) \]

\[ \int_0^1 e^{-rt} y dt = y \left[ \frac{e^{-rt}}{-r} \right]_0^1 = y \left( \frac{e^{-r} - 1}{-r} \right) = \frac{y}{r} (1 - e^{-rt}) \]

\[ y \rightarrow y - \theta \]

\[ k \rightarrow k + \frac{\alpha}{\alpha - 1} (1 - e^{-rt}) \]

\[ c \rightarrow c \]

\[ y \rightarrow y \]

\[ \alpha \rightarrow \alpha \]
Econ 609 (lec)

Result from last time:
\[ V^t(k, x) = K^{-t} V^1(1, x) \]

Substitute into Bellman eqn.
- Kevin: impact - if it doesn't cancel out, you are under a mistake. Moreover

Do it

FOC - pretty easy
\[ \frac{\partial V^t}{\partial x} = k x \]

Prove the symmetry of \( V \)
\[ V(k, x) = V(x, k) \]

(a) Symmetry of \( V \) w.r.t. interior variables:
\[ (k, x) \in \mathbb{R} \implies (T(k, k), T(x, x)) \in \mathbb{R} \]

(b) Symmetry of the intertemporal constraints (transition function):
\[ S(T(k, k), T(x, x)) = T(k, k) = T(x, x) \]

(c) Symmetry of Bellman associated w.r.t. intertemporal function:
\[ \text{verify that } \psi = \psi(k, x) \]

3. Verify interior optimality:
\[ n \psi_t = \psi_t(k, x, \psi_t) \]

4. Prove \( V \) satisfies interior optimality:
\[ \psi_t = \psi_t(k, x, \psi_t) \]

5. Prove \( V \) satisfies Bellman optimality:
\[ V(T(k, x)) = S(V(k, x), k) \]

6. Prove \( V \) satisfies transversality:
\[ V^t(k, x) = 0, \quad V^t(1, x) = 0 \]

For the new realization, go back to the construction and make sure \( k \) is good.

1. Verify interior optimality:
\[ \psi_t = \psi_t(k, x) \]

2. Verify Bellman optimality:
\[ V^t(k, x) = S^t(V(k, x), k) \]

3. Verify transversality:
\[ V^t(1, x) = 0 \]

4. Prove \( V \) satisfies transversality:
\[ V^t(k, x) = 0 \]

5. Prove \( V \) satisfies Bellman optimality:
\[ V^t(k, x) = S^t(V(k, x), k) \]

6. Prove \( V \) satisfies transversality:
\[ V^t(1, x) = 0 \]

7. Prove \( V \) satisfies Bellman optimality:
\[ V^t(k, x) = S^t(V(k, x), k) \]

8. Prove \( V \) satisfies transversality:
\[ V^t(1, x) = 0 \]

9. Prove \( V \) satisfies Bellman optimality:
\[ V^t(k, x) = S^t(V(k, x), k) \]

10. Prove \( V \) satisfies transversality:
\[ V^t(1, x) = 0 \]

Backward induction - it works at \( T \) - see next candidate. Back, satisfying the constraints. The candidate at any \( t \) will be there.
Post Strategy:
(i) Show \( V(T_k(k)) - S(V(k), k) \geq 0 \)
(ii) Show that \( V(T_k(k)) - S(V(k), k) \leq 0 \).
(iii) \( S(V(T_k^{-1}(k)), T_k^{-1}(k)) - V(k) \geq 0 \).

As a consequence of symmetry:
\[ V^{*}(k, T) = \frac{1 - e^{-\rho(t-T)}}{\rho} + V(k, T) \]

In particular,
\[ V^{*}(1, T) = \frac{1 - e^{-\rho(t-T)}}{\rho} + V^{*}(k, T) \]
\[ \Rightarrow V^{*}(k, T) = V^{*}(1, T) - \rho k \left[ 1 - e^{-\rho(t-T)} \right] \]

Proof of (iii):
\[ S(V(T_k^{-1}(k)), T_k^{-1}(k)) - V(k) \]
\[ = \sum_{x} \max_{x} \left[ \Psi(T_k^{-1}(k), x, E[V^{**}[\Gamma(T_k^{-1}(k), x, \omega_{k, m})]], T_k^{-1}(k)) \right] 
- \max_{x} \left[ \Psi(k, x, E[V^{**}[\Gamma(k, x, \omega_{k, m})]], T_k^{-1}(k)) \right] \]

\[ \geq \sum_{x} \left[ \Psi(T_k^{-1}(k), x, E[V^{**}[\Gamma(T_k^{-1}(k), T_k^{-1}(k), \omega_{k, m})]], T_k^{-1}(k)) \right] 
- \Psi(k, x, E[V^{**}[\Gamma(k, x, \omega_{k, m})]], T_k^{-1}(k)) \]

Use an argument of \( \nu \cdot \Phi(k, x, \omega_{k, m}) \)
\[ \geq \sum_{x} \left[ \Psi(T_k^{-1}(k), x, E[V^{**}[\Gamma(T_k^{-1}(k), T_k^{-1}(k), \omega_{k, m})]], T_k^{-1}(k)) \right] 
- \Psi(k, x, E[V^{**}[\Gamma(k, x, \omega_{k, m})]], T_k^{-1}(k)) \]
\[ \geq 0 \quad \text{by (i)} \]

Need \( \Psi \) in 3rd argument.
\[
\text{Symmetry: } \Psi [T(x), T(k, x', \omega)] = \text{Sc}(V(\Gamma(T(x), T(k, x', \omega))))
\]

\[
\text{Precision property: } \leq \max \{ \Psi[T(x), 0, \omega], \text{Sc}(\Gamma(T(x), x, \omega)) \}
\]

\[
= V(T(x, \omega))
\]

\[
\text{Since } V(x) \leq S(V(\Gamma(T(x, \omega)), T(x, \omega))) \text{ if } x \text{ is valid in } T(x)
\]

\[
V(T(x)) \leq S(V(\Gamma(T(\omega), \omega)))
\]

and we can \( S(V(x), \omega) = V(T(x, \omega)) \)

\[
\therefore \ (V(x), \omega) = V(T(x, \omega))
\]

**Example:** \( \int_0^\infty e^{-\alpha x} dx \)

\( C \rightarrow C + \theta \)

\( W \rightarrow W + \alpha \text{ Poisson source} \)

\( V \rightarrow e^{-\alpha V} \)

\[
\text{Now } V(\text{new } \omega) = e^{-\alpha} V(W, \omega)
\]

\[
Q = -W \rightarrow V(0, \omega) = e^{+\alpha} V(W, \omega)
\]

\[
V(0, \omega) = e^{+\alpha} V(W, \omega)
\]

\[
\text{Max viewed new } P
\]

\[
\text{less viewed at high } P
\]
Examples of Symmetries: Th - an old website

Types of Symmetries:

1. Scale Symmetry
2. Capitalization Symmetry
3. Time Invariance - Very Subtle
4. Rule of the Symmetry - "Almost always true, the case"
5. Additive Symmetry (groupwise utility) - not unique and the theory of games.

\[ V(\alpha x_i) = \alpha V(x_i) \] (1) \( k \rightarrow \mathbb{R}^+ \) ? Scaling \( k \), \( x \) are vectors, can also work with scale?

\[ x \rightarrow \alpha x \]

\[ \frac{d}{dt} \alpha = \frac{1}{2} \]\n
\[ V(x) = a(x, p, t) \] (2) \( k \rightarrow (k \cdot t) \)

\( V(L) : V(k) \)

\[ V(x) \rightarrow V' \), in the language of the enlarged \( \mathbb{C}^1 \), so \( u(x) \) undergoes

(3) Riccardi Equivalence - don't change BC

Hartman - Miller Th

1. \( t \rightarrow t + \Theta \) Not changing anything, just passing

\[ T \rightarrow T + \Theta \] they later forward - delaying time

\[ k \rightarrow k \]

\[ x \rightarrow x \]

That is \( \hat{X}_{t+\Theta} - X_t \); \( k_{t+\Theta} - k_t \)

\[ V^t(k, t+\Theta) = V^t(k, T) \]

(4) \( \beta \rightarrow \delta \rightarrow \beta \) \( \beta \rightarrow \beta \rightarrow \beta \) \( \beta \rightarrow \beta \rightarrow \beta \)

\[ \hat{C}(t) = \beta \alpha C(t) \]

PTO \rightarrow
Making anything happen here: 

\[ \hat{\chi}(t) = \chi(t + \frac{t - \hat{t}}{\hat{c}}) \]

\[ \hat{r}(t) = \hat{r}(t + \frac{t - \hat{t}}{\hat{c}}) \]

\[ \chi \rightarrow \hat{\chi} \rightarrow \text{different thing could happen if you're changing it, and} \]

\[ \text{there always exist as a} \]

Dynamic path setting rules: Redlining at the panels space

Types of Structure

- Additive separability (Modularity)
  
  \[ \max_{k_1, k_2} F_1(k_1, x) + F_2(k_2, x) = \max_{k_1} F_1(k_1, x) + \max_{k_2} F_2(k_2, x) \]

- Optimization subject

  \[ V = \max_{k, \lambda} \left( H(k, \lambda) - C(k, x) + EV(x) \right) \]

- End Equations: Speed if less leading. 

  \[ \begin{aligned}
  \text{speed the \text{th} way. Be the new \text{speed} of B/C'}
  \end{aligned} \]

- Unique: Being up value \( \hat{t} \)

\[ \text{let's get something up again: beam up value \( \hat{c} \)} \]

- If you started out 

  \[ \text{optimal, cannot be better of this change, it's right at the mean.} \]

  \[ \text{Theorem:} \]

  \[ \hat{k}_i = \hat{k}_i \text{ but } \text{Not } \hat{k}_i \text{, both.} \]

\[ \text{No in the details with speed that not control how fast to squeeze a} \]

a finite amount of time. 

\[ \text{correlation out of the need to get habit.} \]

\[ \text{Habit Formation}: \text{UCC}(1, H); H = \rho(C-H) \text{, and control } \text{UCC}(1, U). \]

\[ \text{a correlation } f^2 \text{ of } H \]

\[ \text{if one correct, change valid, but then you can never} \]

\[ \text{it.} \]
Example: $\text{UCC}_t = \text{HH}_t$ (constant)

Wealth: $\text{HH}_t = \text{HH}_t + \frac{1}{2} \text{HH}_{t-1} - \frac{1}{2} \text{HH}_t$ (ovens can be used to shape the economy)

Spend $\varepsilon$ less today, spend $\varepsilon + \frac{1}{2} \varepsilon$ more, put consumption back on track.

- Changed habit

$\text{HH}_t = \text{HH}_t + \frac{1}{2} \varepsilon$

Wealth track:

\[
\begin{align*}
\text{WRR}_t &= \text{HH}_t + \frac{1}{2} \varepsilon \\
\text{WRR}_{t+1} &= \text{WRR}_t - \frac{1}{2} \varepsilon (\text{if I spend the extra money})
\end{align*}
\]

$\text{HH}_t = (1 - \frac{1}{2}) \left( \frac{1}{2} \text{HH}_t + \frac{1}{2} \varepsilon \right) + \frac{1}{2} \left( \text{HH}_{t-1} - \varepsilon \right)$

\[
\begin{align*}
\text{HH}_t &= \text{HH}_t + \frac{1}{2} \varepsilon \\
\text{HH}_{t+1} &= (1 - \frac{1}{2}) \varepsilon (\text{HH}_t - \frac{1}{2} \varepsilon)
\end{align*}
\]

So, change consumption:

\[
\begin{align*}
\text{HH}_t &= \text{HH}_t + \varepsilon \\
\text{HH}_{t+1} &= (1 - \frac{1}{2}) \varepsilon (\text{HH}_t - \frac{1}{2} \varepsilon)
\end{align*}
\]

\[
\begin{align*}
\text{HH}_t &= \text{HH}_t + \varepsilon \\
\text{HH}_{t+1} &= (1 - \frac{1}{2}) \varepsilon (\text{HH}_t - \frac{1}{2} \varepsilon)
\end{align*}
\]

Wealth will now have an increased track, but $\text{WRR}_{t+1} = \text{WRR}_t$.

$\frac{1}{2} \varepsilon$ we also get $\text{HH}_{t+1} = \text{HH}_{t+1}$ - habit bad control

The get even if not controlled

Fudger Eq:

\[
\text{HH}_{t+1} - \text{HH}_t = \text{HH}_{t+1} - \text{HH}_t
\]

\[
\text{HH}_{t+1} - \text{HH}_t = \text{HH}_{t+1} - \text{HH}_t
\]

\[
\text{HH}_{t+1} - \text{HH}_t = \text{HH}_{t+1} - \text{HH}_t
\]

\[
\text{HH}_{t+1} - \text{HH}_t = \text{HH}_{t+1} - \text{HH}_t
\]

\[
\text{HH}_{t+1} - \text{HH}_t = \text{HH}_{t+1} - \text{HH}_t
\]
Econ 609 (lec)

13/02/2008

Error in habit formation - logit utility discount factor.

\[ \max \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) \]

\[ H_t = \beta H_{t-1} + G_t \]

Where \[ G_t \] is a probability forecast

\[ H_t = e^\gamma \frac{1 - p}{p} H_{t-1} \]

Find and demonstrate an Euler eq. with a finite number of terms different from a control theory.

Problem #2: In the same context where I prove that \[ R_t = 0 \] is reasonable, prove that relative prudence \[ f_t \] is preserved (cannot be penalized)

\[ \max \bar{Y} = \phi_1 (U_t, \phi_2 (H_t, E_t)) \]

\[ \bar{Y} = \phi_3 (e, L_t, \phi_4 (U_t, \phi_5 (H_t, E_t))) \]

5 hours

Optimizing subproblems:

Necessary condition \[ \max_{h_t, e_t} \phi_2 (H_t, E_t) \]

Weak separability: \( \frac{\partial}{\partial H_t} \phi_2(H_t, E_t) = 0 \)

You find that \( E_t \) can't come over - neverForecast. \( E_t \) is a function of \( H_t \) have

Optimizing subproblems give you insight into the original problem.

Another: \[ \min_{e_t} \phi_1 (U_t, \phi_2 (H_t, E_t)) \]

\( \phi_4 (U_t, \phi_5 (H_t, E_t)) = \text{constant} \)

Single step: \( U_t, H_t, E_t = U(H) \)

Weak separability: \( \frac{\partial}{\partial H_t} \phi_2(H_t, E_t) \neq 0 \)

It is never suppressed by itself - just wax out over it and can almost get rid of it.
If we assume that the original problem is to minimize the cost of a communication network, we can write:

\[ \text{Cost} = \sum_{i,j} c_{ij} x_{ij} \]

where \( c_{ij} \) is the cost of connecting nodes \( i \) and \( j \), and \( x_{ij} \) is a binary variable indicating whether a link is included in the network.

To find the optimal solution, we can use a linear programming approach. The constraints would include:

1. \( \sum_{j \neq i} x_{ij} = 1 \) for each node \( i \) (each node must be connected to exactly one other node).
2. \( x_{ij} \in \{0, 1\} \) for all \( i, j \).

By solving this linear program, we can find the minimum cost network that satisfies the connectivity requirements.

Once the network is designed, we can analyze its performance and make adjustments as necessary.
\[
V(k) - \lambda k = u(c) - \lambda c + \sum_i p_i(\xi_i) - \lambda \xi_i \\
\max_{k} V(k) - \lambda k = \max_{c} u(c) - \lambda c + \sum_i \max_{\xi_i} p_i(\xi_i) - \lambda \xi_i \\
\text{Conjugate} \\
\rightarrow -V^*(k) \\
\rightarrow -u^*(c) + \sum_i \xi_i^* = -z^*(\lambda) \\
\text{Conjugate} \text{ } k^{th} \text{ add } u^* \\
\text{Vertical additivity: if } V \text{ is proper, so is } V + u \text{ to } j \text{ th } V_j \text{ is proper, it adds twice.} \\
V^*(\lambda) = u^*(c) + \sum_i \xi_i^* (\lambda) \text{ (Kernel, Added)} \\
\text{Conjugate } k^{th} \text{ add } u^* \\
\text{What if } u^* \text{ corresponds to } V(c) \text{ and } z(\lambda) \\
V^*(\lambda) = 2V^*(\lambda) - V(V^*(\lambda)) \text{ implies } \lambda = \frac{1}{\text{Var}(u^*(\lambda))} \\
\text{Inverse } k^{th} \text{ Theorem} \\
V^*(\lambda) = V^*(\lambda) + \frac{1}{\text{Var}(u^*(\lambda))} \\
\text{Inverse } k^{th} \text{ Theorem} \\
\therefore V^*(\lambda) = \lambda = V^*(\lambda) \\
v^*(\lambda) = 2V^*(\lambda) - V(V^*(\lambda)) \text{ } \{\text{symmetric}\} \\
\therefore V^*(\lambda) = V^*(\lambda) \text{ and } u^*(\lambda) \\
\text{Two elements in table: without } V, u \text{ and change } k, \lambda \\
\text{Intersect } V \text{ and } V^*
\[ V_s^e(2) + k = V_{in}(2) \]
\[ V_{23}(2) = \frac{1}{V_{in}(V_{in}(2))} \]  
\( (5) \)

Set \( \theta \) - What property does produce look like in \( V \) - does this add up?

**RLA Example:**

\[ \frac{k}{V_{in}} \geq \gamma \]

Look at table - \( k = -V_s^e(2) \), \( V_{in} = \frac{1}{V_{in}} \), \( V_{in} = 1 \)

\[ -\frac{V_s^e(2)}{V_{in}(2)} \geq \gamma \]

\( \text{Use } k > 0 \)

\[ V_{in} \geq \text{ RLA } \geq \gamma \]

For \( V \) to \( V_{in} \), RLA \( \leq \gamma \)

\[ \text{Example: Converting } V_{in} \leq 0 \]

\[ \Rightarrow \frac{1}{V_{in}} \leq 0 \Rightarrow V_{in} \leq 0 \]

If to thing itself become the conjugate 1/1 convert.
Conjugate $q^c = q^2$

Conjugate Pads

\[ \text{ECOM}_h^q (k_0) \]

To prove: $V^h e P$ then $V^h e P^*$ and $V^h e P^*$. 

- If $r^h e P$ then $r^h e P^*$ 
- If $r^h e P^*$ then $V^h(r) = \overline{V^h(r)} = V^h(r^*), r^* e P^*$ (Vertical aggregation)

Horizontal Addition

Conjugate $P^*$: Add up easily - just addition. Conjugate the problem, $V$ is preserved under addition.

- Head $V(\cdot)$ concave

Example: E.H. 

Concavity $V_{k}(k) \leq 0$

\[ V_{\lambda}(\lambda) \leq 0 \]

And concavity preserved under addition $\Rightarrow$ concavity

\[ \frac{kV_k}{V} \geq \gamma \Rightarrow \frac{V}{kV_k} \leq \frac{1}{\gamma} \Rightarrow \frac{\lambda V_k - V_k(\lambda)}{V_k(\lambda)} \leq \frac{1}{\gamma} \]

\[ \Rightarrow \frac{\lambda - V_k(\lambda)}{V_k(\lambda)} \leq \frac{1}{\gamma} \]

\[ \Rightarrow \gamma \geq \frac{V_k(\lambda)}{V_k(\lambda)} \]

Thus, all we need is this thing to add...
\[(1 - \frac{1}{3}) V_3^*(1) \leq V^*(1)\]

**Addition problem**  
\[(1 - \frac{1}{3}) u^*_1(1) \leq u^*(1)\]

\[\frac{1}{3} \left[ -\frac{1}{3} \right] V_3^*(1) \leq V^*(1)\]

**Sum of elements vs. actual value of sum**

If \( u^1 + u^2 = u^* \)

\[\frac{u_2^* + u_3^*}{u_1^*} = V_3^*\]

**Sum to** \( \frac{u_2^*}{u_1^*} \leq \frac{8}{9}\)

\[-k \frac{V_{44}^*}{V_{44}} \leq \frac{2}{9}\]

\[-k u_2^* \leq \frac{2}{9}\]

\[\frac{-k u_2^*}{\frac{1}{2} \bar{e}_2} = \frac{-k u_2^*}{\bar{e}_2}\]

\[\frac{-k \bar{u}_2 + \bar{e}_2}{\frac{1}{2} \bar{e}_2} \leq \frac{8}{9} \left[ \bar{u}_2 + \bar{e}_2 \right]\]

\[-\bar{u}_2 \leq \frac{8}{9} \bar{u}_2\]

\[-\bar{V}_2 \leq \frac{8}{9} \bar{V}_2\]

\[-\frac{k V_{44}^*}{V_{44}} \leq \frac{2}{9}\]

\[-\frac{k V_{44}^*}{V_{44}} \leq \frac{2}{9}\]

\[\Rightarrow \frac{V^*}{V_{44}} \geq \frac{2}{9}\]

\[
\text{Made up so that} \\
\text{would not arise. Join in - commonly used reg det} M \text{ in fact}
\]

**This matrix**

\[
\begin{bmatrix}
U_{33} & U_{34} \\
U_{43} & U_{44}
\end{bmatrix}
\]

\[\text{Kend diagonal \& , and det \( \Theta \rightarrow \infty \)}

\[\text{Act: } U_{33} U_{44} - 0 \neq 20 \text{ is same as property (4)}\]
5) \( \text{Value} \geq 0 \) \( \iff \frac{-V_{ij}}{V_{ii}} \geq 0 \implies V_{ij} \leq 0 \)

6) \( \text{Value} \leq 0 \) 

\[ \text{Conjugate symmetry property of } k' \]

\[ \frac{u''u}{\langle u'' \rangle} = 0 \]

What about \( \mathcal{E} \)?

Show that if \( \frac{-V'_{kk}}{V_{kk}} > \frac{\delta}{k} \), then

\[ V'_{kk} = E_{k} V_k(k+\mathcal{E}) \]

\[ \frac{\delta}{k} \]

Need \( E(\mathcal{E}) = 0 \)

\[ \text{if } \frac{-V_{kk}}{V_{kk}} = \frac{x}{k} \text{ then } -V_{kk}(k+\mathcal{E}) = \frac{x}{k} < \frac{\delta}{k} \]

Define \( M(\mathcal{E}) = \frac{V_{kk}(k+\mathcal{E})}{E_{k} V_{kk}(k+\mathcal{E})} \)  

\[ A(\mathcal{E}) = \frac{V_{kk}(k+\mathcal{E})}{V_{kk}(k+\mathcal{E})} \]
\[ \frac{\delta P_{\text{lib}}(k)}{P_{\text{lib}}(k)} = -\frac{E_{\text{e}}\, V_{\text{u}}(k+\delta)}{E_{\text{e}}\, V_{\text{u}}(k)} \cdot \frac{E_{\text{ee}}}{E_{\text{ee}}\, V_{\text{u}}(k)} \]

\[ \times \frac{M(c)}{M(c)} \]

\[ = \frac{E_{\text{e}}}{E_{\text{e}}\, V_{\text{u}}(k+\delta)} \]

\[ \left( \frac{V_{\text{u}}(k+\delta) - V_{\text{u}}(k)}{V_{\text{u}}(k+\delta)} \right) \]

\[ \frac{E_{\text{e}}(M(c)A(c))}{E_{\text{e}}(M(c)\frac{\delta}{\text{net}})} \]

\[ \geq E_{\text{e}}(M(c)\frac{\delta}{\text{net}}) \]

\[ \frac{\text{dt}}{\text{dt}} = \frac{E_{\text{e}}(M(c)A(c))}{E_{\text{e}}(M(c)\frac{\delta}{\text{net}}) + C_{\text{lin}}(M(c), \frac{\delta}{\text{net}})} \]

\[ \frac{M(c)}{M(c)} \geq \frac{E_{\text{e}}}{E_{\text{e}}(\frac{\delta}{\text{net}})} \]

\[ \frac{\text{dt}}{\text{dt}} \geq \frac{E_{\text{e}}}{E_{\text{e}}(\frac{\delta}{\text{net}})} \]

\[ d_{\text{g}}(\text{c}) > 0 \]

Just add or no demonstration for preference

So now we've shown:

\[ \Delta V_{\text{u}}(c_{\text{m}}) = \beta_{\text{m}} E_{\text{e}} V_{\text{u}}(k + \delta) \]

\[ \frac{E_{\text{e}}\, V_{\text{u}}(k + \delta)}{E_{\text{e}}\, V_{\text{u}}(k)} \]

\[ \geq \frac{\delta}{k_{\text{m}}^2} \]

\[ \Delta V_{\text{u}}(c_{\text{m}}) = \beta_{\text{m}} E_{\text{e}} V_{\text{u}}(k + \delta) \]

\[ \Delta V_{\text{u}}(c_{\text{m}}) = \beta_{\text{m}} E_{\text{e}} V_{\text{u}}(k + \delta) \]

\[ \Delta V_{\text{u}}(c_{\text{m}}) = \beta_{\text{m}} E_{\text{e}} V_{\text{u}}(k + \delta) \]

\[ \Delta V_{\text{u}}(c_{\text{m}}) = \beta_{\text{m}} E_{\text{e}} V_{\text{u}}(k + \delta) \]

\[ \frac{\text{dt}}{\text{dt}} \geq \frac{E_{\text{e}}}{E_{\text{e}}(\frac{\delta}{\text{net}})} \]

\[ \frac{\text{dt}}{\text{dt}} \geq \frac{E_{\text{e}}}{E_{\text{e}}(\frac{\delta}{\text{net}})} \]

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\[ \frac{\text{dt}}{\text{dt}} \geq \frac{E_{\text{e}}}{E_{\text{e}}(\frac{\delta}{\text{net}})} \]
5) \[ V_{\text{in}} > 0 \quad \Rightarrow \quad \frac{V_{\text{in}}}{V_{\text{th}}} > 0 \quad \Rightarrow \quad V_{\text{out}} > 0 \] Let 

6) \[ \frac{V_{\text{in}}}{V_{\text{th}}} > 0 \quad \text{(note since } V_{\text{in}} = V_{\text{conjugate}} \text{ property of } V_{\text{th}}) \] 

and \[ \frac{u^* u}{(u^*)^2} = 0 \quad \text{then you have a power consumption} \] 

\[ \Rightarrow \quad \frac{3V_{\text{in}}}{V_{\text{th}}} > 0 \quad \text{or} \quad \frac{-2V_{\text{in}}}{V_{\text{th}}} = 0 \]

What about \( \text{II} \)?

Show that if \( \frac{-V_{\text{in}}(k)}{V_{\text{in}}(k)} > \frac{Y}{k} \) then

\[ V_{\text{in}}(k) = E_{0} V(k+3) \quad \text{also} \quad \frac{-V_{\text{in}}(k)}{V_{\text{in}}(k)} > \frac{Y}{k} \]

Hence \( E(3) = 0 \)

\[ \frac{-V_{\text{in}}(k)}{V_{\text{in}}(k)} = \frac{Y}{k} \quad \text{then} \quad \frac{-V_{\text{in}}(k+3)}{V_{\text{in}}(k+3)} = \frac{E}{k} < \frac{Y}{k} \]

Define \( M(E) = \frac{V_{\text{in}}(k+3)}{E_{0} V_{\text{in}}(k+3)} \) (MV of the function at k is average MV)

\[ A(E) = \frac{-V_{\text{in}}(k+3)}{V_{\text{in}}(k+3)} \]
$$\lambda = V_{k}(x)$$

$$d\lambda = V_{k}^{'} \, dk$$

$$dk = [V_{kk}]^{-1} \, d\lambda$$ \hspace{1cm} (1)

$$V_{k}^{'} = V_{k}^{'} \, dk$$ \hspace{1cm} (2)

$$V_{k}^{'} \, d\lambda = [V_{kk}]^{-1} \, \lambda$$

$$V_{k}^{'} = [V_{kk}]^{-1} (V_{k}^{-1}(\lambda))$$

$$dV_{k}^{'} = V_{k}^{'} \, dV_{k}^{'}$$

$$V_{kk} V_{k}^{'} = I$$

$$V_{kk} = \frac{1}{2} [V_{kk}]^{-1}$$

$$(dV_{kk}) [V_{kk}]^{-1} + V_{kk} \, d[V_{kk}]^{-1} = 0$$

$$V_{kk} \, d[V_{kk}]^{-1} = -dV_{kk} [V_{kk}]^{-1}$$

$$d[V_{kk}]^{-1} = -[V_{kk}]^{-1} \frac{dV_{kk}}{dk} [V_{kk}]^{-1}$$

$$\frac{d[V_{kk}]^{-1}}{dk} = -[V_{kk}]^{-1} \frac{dV_{kk}}{dk} [V_{kk}]^{-1}$$

$$\sum_{i} - [V_{kk}]^{-1} \frac{dV_{kk}}{dk} [V_{kk}]^{-1} \, dk_{i}$$ \hspace{1cm} (3)

(1) + (2) gets (1.2)

$$dk_{i} = \sum_{j} [V_{kk}]_{ij} \, d\lambda_{j}$$
\[ \text{arbitrary: } V(k, \rho) \]

\[ \lambda = V_k(k, \rho) \quad \Rightarrow \quad k = V_k^{-1}(\lambda, \rho) \]

\[ V(\lambda, \rho) = k V_k \quad - \quad V \]

\[ = V_k^{-1}(\lambda, \rho) \cdot V_k(V_k^{-1}(\lambda, \rho), \rho) - V(V_k^{-1}(\lambda, \rho), \rho) \]

\[ \lambda = V_k(k, \rho) \Rightarrow \quad \frac{d\lambda}{d\rho} = \frac{V_k dk + V_k dp}{V_k} \]

\[ = \frac{V_k dk}{V_k} = \frac{V_k dk}{V_k} = \frac{\lambda}{V_k} \frac{d\lambda}{dp} \]

\[ k = V_k^{-1}(\lambda, \rho) \]

\[ \frac{dk}{d\lambda} = \frac{1}{V_k(V_k^{-1}(\lambda, \rho), \rho)} \]

\[ \frac{d\lambda}{dp} = \frac{V_k}{V_k(V_k^{-1}(\lambda, \rho), \rho)} \]

\[ \lambda = V_k^{-1}(\lambda, \rho) \]

\[ V_k^{-1}(\lambda, \rho) = \frac{1}{V_k} \left( k(V_k - V) \right) \quad \Rightarrow \quad \lambda = \frac{1}{V_k} (kV_k - V) \]

\[ = \frac{kV_k dk}{V_k} + \frac{[kV_k - V] dp}{V_k} \]

\[ = kV_k \left( \frac{1}{V_k} \frac{d\lambda}{dp} \right) \]

\[ = k \left( \frac{d\lambda}{dp} - V dp \right) \frac{dV_k}{d\lambda} \]

\[ \frac{d\lambda}{dp} = \frac{kV_k - V} {V_k} \quad \frac{dV_k}{d\lambda} \]
Using the fact that upward shift is also a rightward shift.

Assume $V_{p_1} > 0$ and $V_{p_2} > 0$.

**Proof:**

1. $V_{p_1} > 0$ implies $\operatorname{Im} p_1 > 0$.

2. \[ V_{p_1} > 0 \Rightarrow \sqrt{V_{p_1}} > 0 \] (Vertical)

3. \[ V_{p_1} > 0 \Rightarrow V_{p_1}^* = -V_{p_1} \]

4. \[ V_{p_1} = \frac{-V_{p_1}}{V_{p_2}} \]

5. \[ V_{p_1} + V_{p_2} \geq 0 \]

Aside: \[ V = U + \Delta V \]

**Final Result:**

\[ V = u \Xi \left( e^{i \frac{\alpha}{2}} + e^{-i \frac{\alpha}{2}} \right) \]

\[ V_{p_1} = -V_{p_1} \left( e^{i \frac{\alpha}{2}} - e^{-i \frac{\alpha}{2}} \right) \]

\[ \frac{\partial V_{p_1}}{\partial e} = -V_{p_1} [ \frac{i}{V_{k+\Xi}} \left( e^{i \frac{\alpha}{2}} - e^{-i \frac{\alpha}{2}} \right) ] - V_{p_1} d\alpha \]

\[ \frac{\partial V_{p_1}}{\partial e} = - \frac{V_{p_1}}{V_{k+\Xi}} + \left( \frac{V_{p_1}}{V_{k+\Xi}} - V_{p_1} \right) d\alpha \]
\[ \text{Concavity of the consumption flow:} \]
\[ c = u_\alpha (\bar{c}), \quad k = v_\alpha (\bar{k}) \]
\[ \frac{dc}{dk} = \frac{u_\alpha'}{v_\alpha'} (\bar{k}) \]
\[ \frac{d^2c}{dk^2} = 0 \]

Read: \[ \frac{d}{d\bar{k}} \left( \frac{dc}{dk} \right) \geq 0 \]

\[ \Rightarrow \frac{d}{d\bar{k}} \left( \frac{\bar{c}^2}{v_\alpha'} \right) = \frac{u_\alpha' \bar{V}_\alpha \bar{V}_\alpha - V_\alpha \bar{V}_\alpha}{(V_\alpha)^2} \geq 0. \]

\[ \Rightarrow \frac{x}{\bar{V}_\alpha} \geq 0 \]

\[ \Rightarrow \frac{V_\alpha'}{V_\alpha} \geq \frac{u_\alpha'}{u_\alpha} \]

\text{Precautionary vs. VaR factor}\]

\[ \text{Conjugate to strategy:} \quad \frac{u_\alpha}{V_\alpha} = 0. \quad \text{(1) (HARA)} \]

\[ \frac{\bar{u}_\alpha}{\bar{V}_\alpha} \geq 0. \quad \text{(2)} \]

Vertical axis: \[ \bar{c} = c \]
\[ \bar{V}_\alpha = V_\alpha \]
\[ \text{Vertical axis:} \]
\[ \bar{c} = c \]
\[ \bar{V}_\alpha = V_\alpha \]
\[ (2) \quad -V_{kh} \left( -\frac{V_{k1h}}{V_{kh}} \right) \quad V_{kh} > 0 \]

\[ \frac{V_{k1h} V_k}{V_{kh}} \geq 0 \]

The matrix is Hermitian.

\[ V_{k1h} = -\Theta V_{kh} \]

\[ V_{k1h} \geq -\Theta V_{kh} \]

\[ V_{kh} > -\Theta V_{kh} \]

\[ V_{kh} = -\Theta V_{kh} \]

\[ V' = V + \Omega \]

\[ \left[ \begin{array}{c} V_{k1h} \\ V_{k1h} \end{array} \right] \rightarrow p.d \quad \text{of the matrix} \]

\[ \text{so the matrix is}\]

\[ \text{Hermitian for } \Theta < 0 \]