**A Symmetry Theorem for Dynamic Stochastic Programming**

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Define:







Assume  for some terminal time .

Theorem: Given the following conditions:

A)  FSD  (RFSD)

B)  (SCC)

C)  (SIC)

D)  (SLU)

Ei)  (IH i)

Eii)  (IH ii)

Then:

 

To demonstrate this result, we prove the following two steps:

i)  

ii)  

Step (i):

Define:



Lemma (L1):

 FSD 

Proof:



 FSD 

Now we write:



 🡨 using (SCC)

🡨 using (SIC) and (SLU)

 🡨 using (IH i), (L1), and (RFSD)

Step (ii):

To prove (ii), it is sufficient to show that

 

Assume that both  and  exist.

Define:



Lemma (L2):



Proof:

Taking  of both sides of the RHS gives us



Now take  and . This yields:



Now we write:



 🡨 using (SCC)

 🡨 using (SIC) and (L2)

 🡨 using (SLU)

 🡨 by (IH ii), (L1), and (RFSD)

This completes the proof of the Generalized Symmetry Theorem.

1. I am deeply grateful to students in my “Advanced Mathematical Methods for Macroeconomics” course at the University of Michigan, particularly Thomas Bridges and Noah Smith, for helping me work out and write out carefully succeeding iterations of this theorem. The text of this iteration is the solution to a final exam problem in Winter 2010 generously provided by Noah Smith. Compare to Thomas Bridges’ lecture notes in “symmetry-theorem.pdf” to see in what sense this exam problem was a generalization. [↑](#footnote-ref--1)