1. Location and Land Use. This problem solves for profit-maximizing choice of labor at 3 distances from the market. Given the profit-maximizing choice of labor, the profit-maximizing levels of tomato output and location rent also are found.

(a) Finding $N^*$. 

Producer A: distance = 4 miles,

Net price = $P - c \cdot x = \$1500/ton - \$200/ton = \$1300/ton.$

Profit-maximization problem:

$$\text{max } 1300 \cdot 4 N \cdot \bar{L}^{0.8} - 20,000 \cdot N$$

$\{N\}$

$$\bar{L} = 160 \text{ acres}$$

$$\frac{d\Pi}{dN} = \frac{1300 \cdot (0.8) \bar{L}^{0.8}}{N^{0.8}} - 20,000 = 0$$

Or,

$$\frac{1300 \cdot 46.4}{20,000} = N^{0.8}$$

Simplifying:

$$3.016 = N^{0.8}$$

and

$$\left(3.016\right)^{\frac{1}{0.8}} = \left(N^{0.8}\right)^{\frac{1}{0.8}} = N$$

Finally,

$$N^* = \left(3.016\right)^{\frac{1}{0.8}} = 3.97 \text{ workers/year.}$$
Producer B: distance = 12 miles

Net price = \( p - \text{c} \cdot x = $1500/\text{ton} - $600/\text{ton} = $900/\text{ton} \).

Profit-maximization problem:

\[
\max_{N^3} \quad 900 \cdot 4 N^2 \bar{E}^{0.8} - 20,000, N
\]

\( \bar{E} = 160 \text{ acres} \).

\[
\frac{\partial \pi}{\partial N} = \frac{900 \cdot 48.4}{20,000} = N^{0.8}
\]

Simplifying and solving:

\[
N^* = \left( \frac{2.088}{0.8} \right)^{1/8} = 2.51 \text{ workers/\text{year}}.
\]

Producer C: distance = 20 miles

Net price = \( p - \text{c} \cdot x = $1500/\text{ton} - $1000/\text{ton} = $500/\text{ton} \).

Profit-maximization problem:

\[
\max_{N^3} \quad 500 \cdot 4 N^2 \bar{E}^{0.8} - 20,000, N
\]

\( \bar{E} = 160 \text{ acres} \)

\[
\frac{\partial \pi}{\partial N} = \frac{500 \cdot 46.4}{20,000} = N^{0.8}
\]

Simplifying and solving:

\[
N^* = \left( \frac{1.016}{0.8} \right)^{1/8} = 1.20 \text{ workers/\text{year}}.
\]
(b) Finding profit-maximizing tomato output and location rent.

Example. Producer A.

Tomato output = \( 4 \cdot (3.97)^2 \cdot (160.8) = 305.6 \) tons/year
Rent = \( 1300 \cdot 305.6 - 20,000 \cdot 3.97 = \$317,844 \) /year

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<thead>
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<th>Summary Table</th>
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<tbody>
<tr>
<td>Producer</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

Example. Producer B.

Tomato output = \( 4 \cdot (2.51)^2 \cdot (160.8) = 278.8 \) tons/year
Rent = \( 900 \cdot 278.8 - 20,000 \cdot 2.51 = \$200,719 \) /year

Example. Producer C.

Tomato output = \( 4 \cdot (1.20)^2 \cdot (160.8) = 240.5 \) tons/year
Rent = \( 500 \cdot 240.5 - 20,000 \cdot 1.20 = \$96,271 \) /year
(c) Do the "main results" hold? Obviously, Yes!

Look at the Summary Table on the previous page.

"Distance to the market" increases as we go from Producer A to B to C.

$N^*$ decrease as distance increases.

Tomato output decreases as distance increases.

Rent decreases as distance increases.

(d) The general form of the bid-rent function is:

Rent decreases in distance.

---

Three points on a bid-rent function.
Initial problem with $\bar{W} = 40$ (40,000 acre-feet per year)

→ (a) Set up and solve Lagrangian.

$$L = -500 + 80 \cdot W_T - 0.5 W_T^2 - 1000 + 110 \cdot W_C - 1.0 W_C^2 + \lambda (40 - W_T - W_C)$$

$$\frac{\partial L}{\partial W_T} = 80 - W_T - \lambda = 0 \Rightarrow 80 - W_T = \lambda$$

$$\frac{\partial L}{\partial W_C} = 110 - 2 W_C - \lambda = 0 \Rightarrow 110 - 2 W_C = \lambda$$

$$\frac{\partial L}{\partial \lambda} = 40 - W_T - W_C = 0 \Rightarrow 40 = W_T + W_C$$

Finding the solution.

- Set: $80 - W_T = 110 - W_C$

- Express the constraint as: $W_T = 40 - W_C$.

- Substitute for $W_T$: $80 - (40 - W_C) = 110 - W_C$

- Simplify: $40 + W_C = 110 - 2 W_C$

$$3 W_C = 70$$

$$W_C^* = 23\frac{1}{3} \left( 23,333.33 \text{ acre-feet per year} \right)$$

$$W_T^* = 16\frac{2}{3} \left( 16,666.67 \text{ acre-feet per year} \right)$$

$$\lambda = 80 - W_T^* = 63\frac{1}{3}$$

(#63.33 per acre-foot)
(b) \( \lambda = \$63.33 \) per W.

Profit in tomato production:
\[
-500 + 80 \cdot 16^{2/3} - 0.5 \left(16^{2/3}\right)^2
\]
\[= \$694,440 \text{ thousand per year}
\]
\[= \$694,440 \text{ per year}
\]

Profit in cotton production:
\[
-1000 + 110 \cdot 23^{1/3} - (23^{1/3})^2
\]
\[= \$1,022,220 \text{ thousand per year}
\]
\[= 1,022,220 \text{ per year}
\]

Mutithread profit = \$1,716,660 per year

(c) Economic interpretation of \( \lambda \).

\[\lambda = \frac{\text{marginal change in profit}}{\text{marginal change in water constraint}}\]

"Profit would increase by \$63.33 following a one acre-foot increase in the water constraint."

(d) Graphs: see next page.
(d) Graphs using consistent scales.

\[ \text{Tomatoes} \]

\[ \frac{\$}{W_T} \]

\[ \lambda = 63.3 \]

\[ 16.3^3 40 80 \]

\[ W_T^* = 43.33 \text{ thousand acre-feet per year} \quad (43,333) \]

\[ \text{Cotton} \]

\[ \frac{\$}{W_c} \]

\[ \lambda = 63.3 \]

\[ 23.3^3 55 \]

\[ W_c^* = 36.67 \text{ per acre-foot} \quad (36,667) \]

(e) Resolve the problem with \( \bar{W} = 80 \).

First order conditions:

\[ \frac{\partial L}{\partial W_T} = 80 - W_T = \lambda \]

\[ \frac{\partial L}{\partial W_c} = 110 - 2W_c = \lambda \]

\[ \frac{\partial L}{\partial \lambda} = 80 = W_T + W_c \]

Solution:

\[ W_T^* = 43.33 \text{ thousand acre-feet per year} \quad (43,333) \]

\[ W_c^* = 36.67 \text{ per acre-foot} \quad (36,667) \]

\[ \lambda = 36.67 \text{ per acre-foot} \]

Profit in tomato production: \$2,027,777 per year

Profit in cotton production: \$1,688,886 per year

Total profit: \$3,716,663 per year
(f) Resolve the problem with \( W = 120 \).

First-order conditions:

\[
\frac{\partial L}{\partial W_T} = 80 - W_T = \lambda \\
\frac{\partial L}{\partial W_c} = 110 - 2 \cdot W_c = \lambda \\
\frac{\partial L}{\partial \lambda} = 120 = W_T + W_c
\]

Solution.

\( W_T^* = 70 \) thousand acre-feet

\( W_c^* = 50 \) thousand acre-feet

\( \lambda = \$10 \) per acre-foot

Profit in tomato production: \( \$2,650,000 \) per year

Profit in cotton production: \( \$2,000,000 \)

Total profit: \( \$4,650,000 \) per year.

(g) Sketch a graph of \( \lambda(W) \).

\[
\begin{array}{c}
\$ / W \\
70 \downarrow \\
50 \\
30 \\
10 \\
\hline
W \\
40 \rightarrow 80 \rightarrow 120
\end{array}
\]

Plot: \((40, 63.33)\), \((80, 36.67)\), \((120, 10)\)
(h) Analytical question. Will a price of $25/acre-foot induce water conservation?

Comparison: \[ 25 > \lambda \Rightarrow \text{conservation} \]
\[ 25 \leq \lambda \Rightarrow \text{no conservation}. \]

From the graph on the prior page, we see that the answer depends on the level of \( \overline{W} \).

**Cases:**

<table>
<thead>
<tr>
<th>( \overline{W} )</th>
<th>( \lambda(\overline{W}) )</th>
<th>Relationship</th>
<th>Conservation?</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>63(\frac{1}{3} )</td>
<td>25 &lt; 63(\frac{1}{3} )</td>
<td>No</td>
</tr>
<tr>
<td>80</td>
<td>36(\frac{2}{3} )</td>
<td>25 &lt; 36(\frac{2}{3} )</td>
<td>No</td>
</tr>
<tr>
<td>120</td>
<td>10</td>
<td>25 &gt; 10</td>
<td>Yes</td>
</tr>
</tbody>
</table>