Transversal switching between generic stabilizer codes

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Introduction

The recently proposed stabilizer rewiring algorithm\textsuperscript{2} (SRA) gives a method for constructing a transversal circuit mapping between any pair of stabilizer codes. We propose a randomized variant of the SRA and show that with at most linear overhead, a path of deformations can be found which preserves the code distance throughout the circuit. This allows constructive, distance-preserving code switching between arbitrary stabilizer error-correcting codes.

Main Result

For any two \([n, k, d]\) stabilizer codes \(S_1\) and \(S_2\), the rSRA scheme gives a transversal circuit mapping from \(S_1\) to \(S_2\) where each intermediate code has distance at least \(d\) with probability \(1 - \varepsilon\), using \(m = O(d \log n/d + \log 1/\varepsilon)\) ancilla qubits.

Simplified Algorithm

For simplicity, we consider mapping between codes \(S, S' : \mathcal{N}(S) \cap S' = \emptyset\) and vice versa.

\textbf{Input:} Generating sets \(G, G'\)

\textbf{Output:} \(G \cup A = G_0, G_1, \ldots, G_{n-k} = G' \cup A'\)

\begin{itemize}
  \item Pick ancilla sets \(A \leftrightarrow \{0\}, A' \leftrightarrow \{+\}\) and extend \(G, G'\) to the correct dimension. More ancilla increases the probability of success.
  \item Choose a random basis \(G_C\) for \((G' \cup A')\).
  \item Choose the unique basis \(G_C\) for \((G \cup A)\) whose elements are canonically conjugate to the elements of \(G_C\).
  \item One-by-one, replace the element of \(G_C\) with their canonically conjugate elements in \(G_C\).
\end{itemize}

Transversal Mapping

For generating sets differing by a single \(g, g'\),

\[
\frac{1}{\sqrt{2}}(1 + g'g)|\psi\rangle_G = |\psi\rangle_{G'}.
\]

Furthermore,

\[
\frac{1}{\sqrt{2}}(1 + g'g)|\psi\rangle_G = \frac{1}{\sqrt{2}}(g' + 1)|\psi\rangle_G = \frac{\sqrt{2}}{2}(1 - g')|\psi\rangle_G.
\]

Thus, the transformations in the final step can be applied transversally by measuring \(g'\) and applying \(g\) conditioned on outcome \(-1\).

Example

<table>
<thead>
<tr>
<th>Type</th>
<th>([7, 1, 3])</th>
<th>([5, 1, 3])</th>
</tr>
</thead>
<tbody>
<tr>
<td>(G_A)</td>
<td>ZZXXZI</td>
<td>YXYXYYIYY</td>
</tr>
<tr>
<td>(G_C)</td>
<td>YXYXZYIYX</td>
<td>XXZXXZI</td>
</tr>
<tr>
<td></td>
<td>XSYXZXIYX</td>
<td>XZXZXXI</td>
</tr>
<tr>
<td></td>
<td>ZXXYIYXIYX</td>
<td>ZXXZXXI</td>
</tr>
<tr>
<td></td>
<td>ZZSYZZIII</td>
<td>ZIYIYX</td>
</tr>
</tbody>
</table>

A distance-preserving path converting from Steane’s \([7, 1, 3]\) code to the perfect \([5, 1, 3]\) code, where \(G_A\) is a basis for \((G) \cap (G')\). In this case, the usual basis for the \([5, 1, 3]\) code suffices. The conversion proceeds from top to bottom, and requires no additional ancilla except to normalize the code sizes.

Discussion

- The algorithm can be extended to \(S, S' : \mathcal{N}(S) \cap S' \neq \emptyset\).
- Practically, the number of required ancilla (if any) is much less than the upper-bound. The required number scales roughly with \(\log(|\mathcal{N}(S) \cap S'|)\).
- Can we boost from distance-preserving to fault-tolerance, possibly by adding error-correction to the ancilla?

References


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