IOE 202: lecture 13 outline

- Announcements
- Last time...
- Making decisions and sequences of decisions under uncertainty

Last time:

- Different types of queueing systems, characterized by
  - Characteristics of arrivals and customer behavior
  - Service discipline
  - Service characteristics
- Steady state analysis of queueing systems with exponentially-distributed interarrival and service times
  - $M/M/1$ system, when $\rho = \frac{\lambda}{\mu} < 1$
  - $M/M/s$ system, when $\rho = \frac{\lambda}{s\mu} < 1$
  - $M/M/s/s$ system
Decision analysis problem: Deciding on summer plans

Bill is an undergraduate student. It is now early in the Fall semester, but Bill is already considering his options for summer employment.

In the end of August, Bill interviewed for a summer internship at a major consulting company. The interview went well, and Bill thinks he has a good chance (about 60%) of getting an offer of an internship with a $14,000 summer salary.

Also, Bill has the opportunity to return to the summer job he held last year. The job will pay $12,000 for the summer. However, the offer to return to this job will only remain open until the end of October, while the internship offers are made in mid-November. Therefore, Bill has to accept or decline the job before he knows if he has an offer of an internship.

If Bill gets offered an internship, he needs to respond by the end of November.

If Bill were to turn down the job, he could either accept the potential internship offer (were it indeed to materialize), or he could look for a different summer job via the University placement service.

Interviews in the placement office begin in December. Bill collected some data on the salaries for summer jobs obtained through the placement office, and he thinks that the percentages in the table are a good approximation of his own chances of getting similar offers.

<table>
<thead>
<tr>
<th>Salary</th>
<th>% of students offered this salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>$21,600</td>
<td>5%</td>
</tr>
<tr>
<td>$16,800</td>
<td>25%</td>
</tr>
<tr>
<td>$12,000</td>
<td>40%</td>
</tr>
<tr>
<td>$6,000</td>
<td>25%</td>
</tr>
<tr>
<td>$0</td>
<td>5%</td>
</tr>
</tbody>
</table>

Bill’s main criterion for differentiating between different options is salary.

What should Bill’s decision be?
What are the decisions Bill is facing

- 
- 
- 
- 

Analyzing and structuring Bill’s decision problem

- Here, Bill is facing a sequence of decisions under uncertainty
- The decisions Bill must make must be considered chronologically
- The decisions Bill will make in the future will depend on what decisions he made in the past, and what additional information he gained since then (i.e., how some of the uncertainties got resolved).
- A convenient way of representing Bill’s decision-making together with the uncertain events that affect it is via a decision tree
Structure of a decision tree

- Decisions/actions are represented by boxes, a.k.a. **decision forks, or decision nodes**
- Each line emanating from a decision fork corresponds to a possible decision/action that can be made at this point; they are called **branches**
- Uncertain events are represented by circles, or **chance forks, or chance nodes**
- Branches emanating from chance nodes describe all possible outcomes of the associated uncertain event; the probability of each outcome is written on the corresponding branch
- "Final" branches of the tree are assigned numerical values, or payoffs, associated with this sequence of decisions and events, based on the decision criterion (in our case, salary)

Bill's decision tree
Using the tree to make a decision

- Note that the path followed to reach each payoff is determined both by the decisions made and by random events outside of the decisionmaker’s control.
- Bayes’ Decision rule: Using the best available estimates of probabilities of the respective uncertain outcomes, calculate the expected value of the payoff for each of the possible sequence of decisions. Choose the decisions with the maximum expected payoff.

Backward induction procedure for Bayes’ Decision rule

- Start the calculations on the right hand side, and move left (i.e., backwards in time)
- Whenever you reach a chance node, calculate the expected payoffs at that point in the process
- Whenever you reach a decision node, choose the decision with the largest expected payoff at that point in the process.
Bill’s optimal strategy

► In October, contact my previous boss and tell him I will not be back next summer
► In November, if offered an internship, accept it
► If not offered an internship, I will pursue the services offered by the job placement service in December

Another decision problem: Scheduling an outdoor event

► You are in charge of organizing and outdoor event as part of the Ann Arbor summer festival, scheduled to take place on June 15th. The earnings from the event will depend heavily on the weather: if it rains on 06/15, the show will lose $20,000; otherwise, i.e., if it is sunny, the show will earn $15,000. Historically, the likelihood of it raining on any given day in mid-June is 27%.

► Today is May 31. You have the option of canceling the event by the end of today, but if you do so, you will then lose your $1,000 deposit on the facilities.

What should you do? Construct your decision tree, and base your answer on Bayes’ decision rule
Suppose that you can also cancel the show on June 14th, but if you do so, you must pay a fee of $10,000. The advantage of waiting until 06/14 is that you can listed to the next-day weather forecast on the local news station before making your final decision.

According to station records, the station’s next-day forecast in mid-June is “sunny” 90% of the time.

When the weather forecast was “sunny,” the next day turned out to actually be sunny 80% of the time; when the weather forecast was “rain,” it actually rained 90% of the time.

Extend the above decision tree to obtain your new strategy.
Weather information for scheduling an outdoor event in mid-June

- According to station records, the station’s next-day forecast in mid-June is “sunny” 90% of the time.
- When the weather forecast was “sunny,” the next day turned out to actually be sunny 80% of the time.
- When the weather forecast was “rain,” it actually rained 90% of the time.
- Historically, the likelihood of it raining on any given day in mid-June is 27%.

Are these probabilities and frequencies consistent with each other?

Two ways of measuring reliability of the forecast:

- How frequently are the station’s forecasts of a sunny day correct? What about rainy days?
- How frequently is a sunny day correctly forecast by the station? What about rainy days?

Forecast reliability

What fraction of June days are sunny?
What fraction of sunny days are correctly forecast?
What fraction of sunny days are forecast as rainy?
What fraction of June days are rainy?
What fraction of rainy days are correctly forecast?
What fraction of rainy days are forecast as sunny?