Physics 390: Homework 8

For full credit, show all your working.

1. **Quantum mechanics of many particles:** The neutron and the neutral pion are two subatomic particles with no charge, so that they neither electrically repel nor attract other particles. Here are their properties:

<table>
<thead>
<tr>
<th>particle</th>
<th>charge</th>
<th>spin</th>
<th>mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>neutron</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>$1836m_e$</td>
</tr>
<tr>
<td>pion</td>
<td>0</td>
<td>0</td>
<td>$264m_e$</td>
</tr>
</tbody>
</table>

where $m_e$ is the electron mass.

(a) Calculate the lowest energy that ten noninteracting neutrons can have if they are trapped in a three-dimensional cubic quantum well of size 10 fm on a side.

(b) Calculate the corresponding energy for ten pions.

For both (a) and (b), explain the steps in your calculations and the effects of the spins of the particles.

2. **The quantum harmonic oscillator at finite temperature:** We previously studied the states of a single particle in a quadratic energy well—the quantum simple harmonic oscillator—finding that it has states of energy $E_n = (n + \frac{1}{2})\hbar\omega$, where $n = 0, 1, 2 \ldots$ is a non-negative integer and $\omega$ is the angular frequency of oscillation of a classical particle in the same potential.

Suppose the well and the particle in it are at a finite temperature $T$. The partition function $Z$ of the system is given by a sum over states $s$ thus:

$$Z = \sum_s e^{-E_s/kT}.$$  

(a) Show that the partition function for the simple harmonic oscillator is

$$Z = \frac{1}{2} \cosh \frac{\hbar\omega}{2kT}.$$  

(b) Hence find the thermal average energy $\langle E \rangle$ of the particle.

(c) If $\omega = 10^{14}$ s$^{-1}$ and $T = 300$ K, what is the probability that the particle will be in the ground state?
3. **Thermal occupation probabilities:** From observations of the spectrum of a certain star it is determined that about one in a million of the hydrogen atoms in the star is in its first excited state, the rest being in the ground state. (Other excited states can, to a good approximation, be ignored.) Allowing for that fact that the first excited state has a degeneracy of 8 (spin-up and spin-down for each of the three $\ell = 1$ states and one $\ell = 0$ state) while the ground state has a degeneracy of only 2 (spin-up and spin-down), what is the temperature of the star?