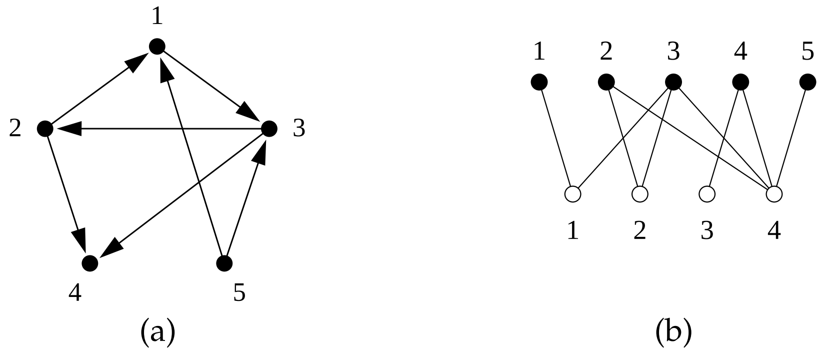
Complex Systems 535/Physics 508: Homework 1

1. Consider the following two networks:



Network (a) is a directed network. Network (b) is undirected but bipartite. Write down:

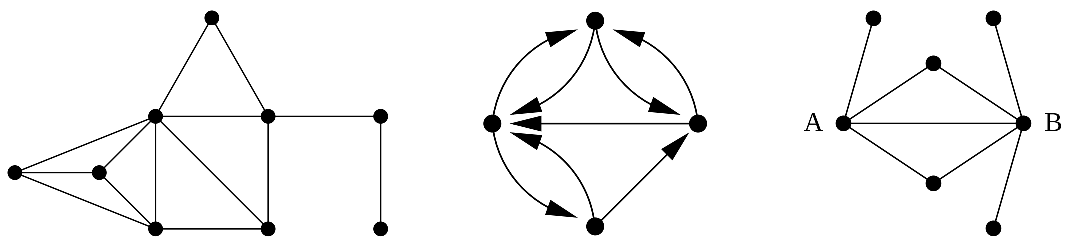
(i) the adjacency matrix of network (a);

(ii) the cocitation matrix of network (a);

(iii) the incidence matrix of network (b);

(iv) the adjacency matrix for the network created when we project network (b) onto its black vertices.

2. Consider these three networks:



(i) Find a 3-core in the first network.

(ii) What is the reciprocity of the second network?

(iii) What is the cosine similarity of vertices A and B in the third network?

3. Consider a k-regular undirected network (i.e., a network in which every vertex has degree k).

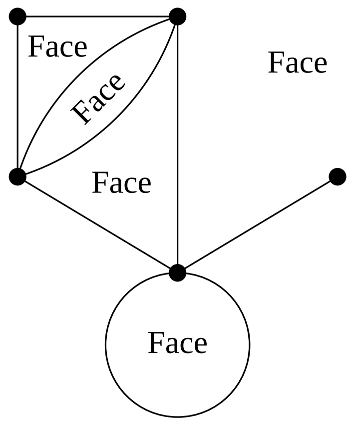
(i) Show that the vector $\mathbf{1} = (1, 1, 1, \ldots)$ is an eigenvector of the adjacency matrix with eigenvalue k.

(ii) By making use of the fact that eigenvectors are orthogonal (or otherwise), show that there is no other eigenvector that has all elements positive. The Perron–Frobenius theorem says that the eigenvector with all elements positive has the largest eigenvalue, and hence the eigenvector $\mathbf{1}$ gives, by definition, the eigenvector centrality of our k-regular network and the centralities are the same for every vertex.

(iii) Find the Katz centralities of all vertices in a k-regular network.

(iv) Name a centrality measure that could give different centralities for different vertices in a regular network.

4. Consider a connected planar network with n vertices and m edges. Let f be the number of "faces" of the network, i.e., areas bounded by edges when the network is drawn in planar form. The "outside" of the network, the area extending to infinity on all sides, is also considered a face. The network can have multiedges and self-edges:



   (i) How do n, m, and f change when we add a single vertex to such a network along with a single edge attaching it to another vertex?

   (ii) How do n, m, and f change when we add a single edge between two extant vertices (or a self-edge attached to just one vertex), in such a way as to maintain the planarity of the network?

   (iii) What are the values of n, m, and f for a network with a single vertex and no edges?

   (iv) Hence by induction prove a general relation between n, m, and f for all connected planar networks.

   (v) Now suppose that our network is simple (i.e., it contains no multiedges or self-edges). Show that the mean degree c of a simple, connected, planar network is strictly less than six.

5. In a survey of couples in the city of San Francisco in 1992, Catania *et al.* recorded, among other things, the ethnicity of interviewees and calculated the fraction of couples whose members were from various ethnic groups. The fractions were as follows:

		Women				
		Black	Hispanic	White	Other	Total
Men	Black	0.258	0.016	0.035	0.013	0.323
	Hispanic	0.012	0.157	0.058	0.019	0.247
	White	0.013	0.023	0.306	0.035	0.377
	Other	0.005	0.007	0.024	0.016	0.053
	Total	0.289	0.204	0.423	0.084	

Assuming the couples interviewed to be a representative sample of the edges in the undirected network of relationships for the community studied, and treating the vertices as being of four types—black, hispanic, white, and other—calculate the numbers e_{rr} and a_r that appear in Eq. (7.76) for each type. Hence calculate the modularity of the network with respect to ethnicity. What do you conclude about homophily in this community?