Physics 390: Homework 1

For full credit, show all your working.

1. Problem 3-8 from Tipler & Llewellyn.

2. Planck’s law: As we saw in class, the average energy of a mode in Planck’s quantized theory of the radiation in a cavity is given by

\[ \bar{E} = \frac{h f \sum_{n=0}^{\infty} n e^{-nhf/kT}}{\sum_{n=0}^{\infty} e^{-nhf/kT}}. \]

(a) Calculate the value of the sum in the denominator by rewriting it as a geometric series \( \sum_{n=0}^{\infty} a^n \) for some value of \( a \) (to be determined) and then performing the sum using the standard formula for a geometric series.

(b) Show that

\[ -\frac{kT}{h} \frac{\partial}{\partial f} \sum_{n=0}^{\infty} e^{-nhf/kT} = \sum_{n=0}^{\infty} ne^{-nhf/kT}. \]

Hence, using the answer to part (a), show that

\[ \sum_{n=0}^{\infty} ne^{-nhf/kT} = \frac{e^{hf/kT}}{(e^{hf/kT} - 1)^2}. \]

(c) Hence derive Planck’s expression for \( \bar{E} \), Eq. (3-17) in the book.

3. Wien’s law: Given Planck’s radiation law,

\[ u(\lambda) = \frac{8\pi hc}{e^{hc/\lambda kT} - 1}, \]

we can derive Wien’s law.

(a) Differentiate to show that the wavelength of maximum radiation \( \lambda_m \) depends on temperature as \( \lambda_m = \frac{b}{T} \) for some constant \( b \).

(b) Find the constant \( b \) to two significant figures and state its units. You will probably need to know that the solution to the equation \( 5e^{-x} + x = 5 \) is 4.965…

4. Problem 3-15 from Tipler & Llewellyn, parts (a) and (c) only.

5. Problem 3-45 from Tipler & Llewellyn.