Complex Systems 899: Homework 1

For full credit, show all your working.

1. **Fixed points:** For each of the following cases, give an equation of the form $\dot{x} = f(x)$ with the stated properties or, if there is no such equation, explain why not:
   (a) Every integer is a stable fixed point.
   (b) Every real number is a fixed point.
   (c) There are precisely three fixed points, and all of them are stable.
   (d) There are no fixed points.
   (e) There are precisely 100 fixed points.

2. **Exact solution of the logistic equation:** Derive the exact solution of the logistic equation $\dot{x} = \beta x(1-x)$, where β is a constant, for the case $x(0) = x_0$ with $x_0 > 0$.

3. **Nonunique solutions:** The equation $\dot{x} = \sqrt{x}$ with initial condition $x(0) = 0$ has a trivial solution $x(t) = 0$ for all t. Find another solution to the same equation with the same boundary. Explain the physical reason why there are two solutions. How would a real system decide which solution to follow?

4. **Linear stability analysis:** Use linear stability analysis to classify the fixed points of the following equations (or show that such an analysis breaks down):
   (a) $\dot{x} = x(1-x)(2-x)$
   (b) $\dot{x} = \tan x$
   (c) $\dot{x} = x^2(4-x)$
   (d) $\dot{x} = \mathrm{e}^{-1/x^2}$
   (e) $\dot{x} = \ln x$

5. **Saddle-node bifucations:** For each of the following equations show that there is a saddle-node bifurcation in the dynamics of x for some value of the parameter r, and determine that value. Sketch the bifurcation diagram of fixed points x^* as a function of r in each case:
   (a) $\dot{x} = r - \cosh x$
   (b) $\dot{x} = x^2 + rx + 1$
   (c) $\dot{x} = r + x - \ln(x+1)$
   (d) $\dot{x} = r^2 + \frac{1}{4}x - x/(1+x)$