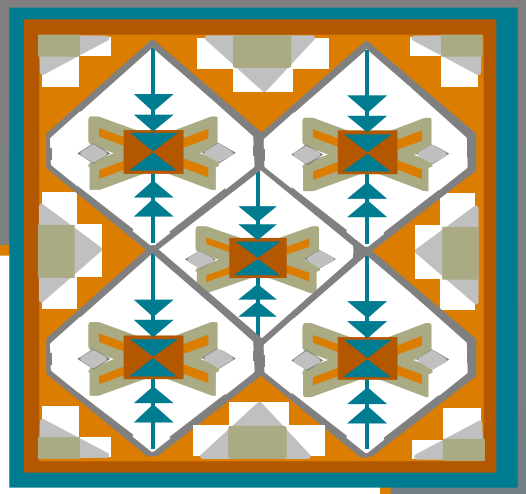


Indigenous Mathematics

Including an In-Depth Look at the Number Theory of the Maya

In Celebration of the Martin Luther King, Jr. Holiday and in Honor of our PhD alumna Marjorie Lee Browne

January 17, 2022

Long Count Date: 13.0.9.3.14
Calendar Round Date: 3 Ix 12 Muan, i.e., (3,14,292)

Bob Megginson
University of Michigan

Happy New Year!

In Lakota, I would say *Omaka teca oiyokipi!*

If you encounter a Maya, say to them the same phrase in the Yucatec Maya language: *Utzul mank'inal!*

Whoa!!! Aren't they extinct???

Not at all!

Living native speakers of all 32 Maya languages combined: ≈ 6,000,000. (Main reference language: Yucatec, about 770,000 native speakers. Quiché has more speakers, over 1,000,000, but the Yucatec branch is about 1,500 years older.)

Living native speakers of all Baltic state languages (Lithuanian, Latvian, Latgalian, Estonian) combined: ≈ 6,000,000

Also, living native speakers of all Nahuatl (Mexica, i.e., Aztecan) languages residing just in Mexico: ≈ 1,700,000. The Mexica would say *Pacuini yancuic xihuitl.*

General Outline of Major Parts

1. Do culture and mathematics influence each other?
2. Distance numbers and the Long Count, the only Maya absolute dating system
   - Answer to this burning question: Did the world end on December 21, 2012? (Spoiler alert: Probably not.)
3. Maya base 20 numbering system
4. Other Maya calendars, all cyclic
   a. Sacred Round or *Tzolkin* (a cycle formed from two cycles running simultaneously, repeating every 260 days)
   b. Vague Year or *Haab* (a 365 day cycle)
   c. Calendar Round (a 52 year cycle based on the Tzolkin and Haab)
5. Date computations
6. Did too much reliance on number theory (calendric computations involving modular arithmetic) contribute to the end of the Mexica empire? (Possibly.)

This presentation will be posted on the Web for folks who want to explore further some of the topics we will briefly visit. See:
http://www-personal.umich.edu/~meggin/IndigenousMath.pdf

Does culture influence mathematics (and conversely)?

From Kitlinermiut (Copper Inuit) oral tradition, a not-so-positive cultural reflection on working with numbers:

Two hunters return, one with a wolf, the other with a caribou. They begin arguing as to which hide has the most hairs, and in order to settle the argument, decide to have a contest, each pulling the hairs out one at a time. They count and count and become so engrossed in what they are doing that days pass and they die of hunger. “That is what happens”, the Inuit storyteller adds, “when one starts to do useless and idle things that can never lead to anything.”

And furthermore …

According to early 20th century accounts, among many Crow it was considered, at that time, bad to try to count above a thousand, since honest people have no use for higher numbers!

Both recounted by the editor in *Native American Mathematics*, Michael Closs, ed., University of Texas Press, 1986, p. 16. (This remarkable book will be cited frequently in what is to follow, and is an essential reference for those interested in the subjects discussed in this presentation.)

WE ONLY LUMP THINGS TOGETHER WITH A COUNT WHEN WE ARE NOT INTERESTED IN KNOWING THEIR INDIVIDUAL IDENTITIES.

Skip to numbering system of the Anishinabeg

The Eastern Cree vs. Hydro-Quebec

Beginning in 1971, the Eastern Cree of Canada fought Hydro-Québec in court because of its James Bay hydroelectric project that would flood expanses of their traditional lands. Their resistance did end up being partially successful.

In one court case, a lawyer for the developers was questioning a Cree hunter appearing as a witness, attempting to show that the hunter did not really know much about a region he claimed was of great importance to him and his people. The lawyer asked, “How many rivers are there in your territory?” The hunter could not answer the question. The lawyer turned in triumph to the judge believing his point was made.

What he, and probably the judge too, did not understand was that this ignorance of the number of rivers was caused by the hunter’s particularly intimate knowledge of his territory. The hunter knew every one of the *many* rivers in his territory individually with all of their different traits, and therefore had no need to lump them all together in his head with a count of how many there were.

(Excerpted with small modifications from “Cultural Ecology of Mathematics”, J. Peter Denny, pp. 132–133 of the edited volume mentioned on the preceding slide.)

The point is that **WE OFTEN LUMP THINGS TOGETHER WITH A COUNT WHEN WE HAVE NO NEED TO KNOW THEIR INDIVIDUAL IDENTITY!**

One related attitude that has influenced number systems used by some indigenous North American peoples, but also appears, I understand, in other numbering systems such as that perhaps of the ancient Egyptians, is that when we have to lump things together with a count, it might be helpful to tell the listener at least a few things about what the objects we are lumping together *do* have in common that justifies the lumping.

Number Words in Anishinaabemowin (language of the Anishinaabeg, or Ojibwe)

Suffixes on number words indicate dimensionality, manipulability, whether or not organic

-aabik, hard and inorganic

midaasw-aabik asiniin, ten-hard.inorganic stones

-minak, 3D organic solids

niizho-minag miinan, two-3D.organic blueberries

-eg, 2D organic solids

niiw-eg ozhashkwayaanag, four-2D.organic muskrat skins

skip

Number Words in Anishinaabemowin (2)

-aatig, 1D organic rigid

ningod-waasw-aatig misan, six-1D.organic.rigid pieces of firewood

-aabiig, 1D organic flexible

naanw-aabiig wadabiin, five-1D.organic.flexible roots

Two special suffixes that act as nouns

-oonag, boats

niizh-oonag, two boats

-gamig, houses

niizho-gamig, two houses

Number Words in Anishinaabemowin (3)

Two cases in which the suffix is omitted

The objects are complex and their classification is unclear

niizh makizinan, two shoes

The object cannot, or *should* not, be manipulated

niswi aanakodoon, three clouds

niizh makoog, two bears

(Examples on this slide and the preceding two are from Denny, pp. 148–149)

Maori Mathematical Sensibility

"...cultural practices including ethnomathematical ones cannot be separated from the language in which they were developed. Changing the language or the linguistic register in which the practices are discussed will have an impact on how the practices are perceived by students. This could result in a loss in the fundamental values that would normally accompany the practices."

- From "The role of language in ethnomathematics", Meaney, Fairhill, Trinick, available on the Web.

Tony is himself Maori and invited me to a traditional Maori midday meal, and told me what they were doing to address this. In collaboration with Bill Barton, and colleagues at the University of Auckland, they gathered Maori ethnomathematicians and teachers together with Maori elders, described certain mathematical terms that had been ported unchanged from English into Maori language elementary school arithmetic texts where instruction was done in the Maori language, and asked the elders what terms *they* would use to describe the object, or property, or action they heard being discussed, such as, perhaps, "divisor". When those terms were substituted into the texts for the English terms, the Maori students did noticeably better, and that success in elementary school mathematics persisted when the students later continued their education at higher levels where instruction was conducted exclusively in English.

An issue requiring rebuttal

Long-standing piece of folklore claiming that something inherent in Western hemisphere indigenous peoples makes us naturally bad at mathematics.

Most toxic manifestation: Papers in reputable neuroscience journals in the 1980s seemingly showing that something in American Indian genetic makeup makes us bad at mathematics (the theory of the “right brained Indian”).

That’s demonstrably nonsense!

And yet another issue that requires rebuttal

Another long-standing piece of folklore: Even if Western Hemisphere indigenous people are able to do mathematics, anything beyond basic counting is culturally irrelevant, and not something we really need in life or in which we would have seen the value, because it really has no value to us. If we are to learn well, this stuff should not be forced upon us.

In the 1930s, in an effort to make the U.S. Bureau of Indian Affairs Uniform Course of Study for the boarding schools more relevant for its students, algebra and geometry were tossed out of the curriculum as culturally irrelevant.

Added to this presentation just this morning

Look in today's *University Record*, or Google:

UM Dearborn Tim Constant Boarding Schools

to get to an article on the boarding schools. The opening lines really struck me:

"*The history of Indian Boarding Schools is probably less well known to most Americans than other historical periods of significant racial oppression and violence, like slavery, Jim Crow, the genocide of indigenous people, and the internment of Japanese Americans during World War II. Even within tribal communities, the topic can be a taboo one*."

In the 1990s I spent almost every summer teaching in enrichment programs at Turtle Mountain Community College, the tribal college of the Turtle Mountain Ojibwe Nation, in North Dakota. I know that there were still memories among the grandparents of many of my students of the Fort Totten boarding school about 90 miles away, which operated until 1939 and is now a state historic site. While I was at the college, there were no educational displays at the college or discussions about the boarding school experience that played such a prominent role in the educational history of that reservation, and the memories were still so raw, particularly among the elders, that it seemed that nobody wanted to talk about it. Never once while I was living and working on that reservation did they. Anyway, moving on….

Homelands of the Maya

Lowlands of Southern Mexico, Guatemala, Belize, western Honduras and El Salvador

Some Definitions

Stele: (pronounced like “steely” or “steel”; the presenter prefers the first pronunciation, for no particularly good reason): An upright stone slab or column typically bearing a commemorative inscription or relief design.

18 Rabbit

Proleptic Gregorian calendar: The modern Gregorian calendar, projected backward through 1582 and to earlier dates as if it had always been used. This is an *absolute dating system*, in which every day is uniquely identified.

CE: Current era or common era, historically called AD

BCE: Before current era; historically called BC

(Photo by Martijn.Munneke, Netherlands)

Distance Numbers

A Mayan almost-base-20 right-to-left place-value numbering system of days, o*nly* used to count days elapsed after some given date, represented *in modern notation* as:

baktuns*.katuns.tuns.uinals.kins

1 kin = 1 day

1 uinal = 20 kins = 20 days

1 tun = 18 uinals = 360 days (sort of a “pseudoyear”)

1 katun = 20 tuns (20 “pseudoyears”) = 7200 days

1 baktun = 20 katuns (400 “pseudoyears”) = 144,000 days

Example (as Maya would say except for the modernized parenthetical comment):

Martin Luther King, Jr. was born on January 15, 1929. The distance number was then 10 kins, 6 uinals, 14 tuns, and 4 katuns (0.4.14.6.10, or just 4.14.6.10), i.e., 94 tuns [“pseudoyears”] plus 130 days) until this presentation on January 17, 2022. King’s 93rd birthday was two days ago.

*The word *baktun* is a modern invention. The other counting words were used far back in history.

Only Absolute Dating System of the Maya: The Long Count

Maya cosmology: After the creation of the world, it took 13 baktuns (about 5125 years) before the world was ready for human habitation. The last day of creation was August 11, 3144 BCE by the proleptic Gregorian calendar (the *Long Count base date*), and the Long Count number was set to 13.0.0.0.0 on that date. Humans showed up the next day, on which the Long Count date was reset to 0.0.0.0.1 (i.e., the “odometer” flipped).

Most dates on Maya monuments run from about 8.12.0.0.0 in about 300 CE to about 10.4.0.0.0 in 900 CE

A converter for translating proleptic Gregorian dates to Long Count can be found at https://maya.nmai.si.edu/calendar/maya-calendar-converter, and one that goes the other direction at https://keisan.casio.com/exec/system/1344494270

What Happens When the Long Count "Odometer" Again Reaches 13.0.0.0.0?

***One "argument"*:** The wise Maya must have understood the total workings of the universe, and knew that the modern world could not last longer than did its creation. Thus, when the calendar would get to 13.0.0.0.0 again, on that day (or the next) the world must end, i.e., we would have reached the end of days.

12.19.19.17.19	(3 Cauac 2 Kankin)*	December 20, 2012 CE
13.0.0.0.0	(4 Ahau 3 Kankin)*	December 21, 2012 CE

Boom!

P.S. Today, January 17, 2022, is 13.0.9.3.14.

P.P.S. There is no evidence that this was ever a religious or cultural tenet of the Maya.

*These are "Calendar Round" dates, and will be explained in a bit.

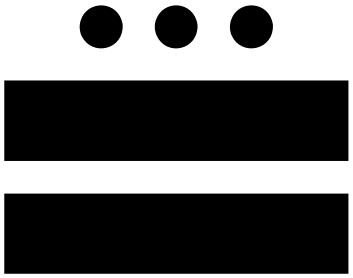
Basic Maya (Possibly Invented by Their Olmec Predecessors) Bar and Dot Numerals 1,2,…,19

1

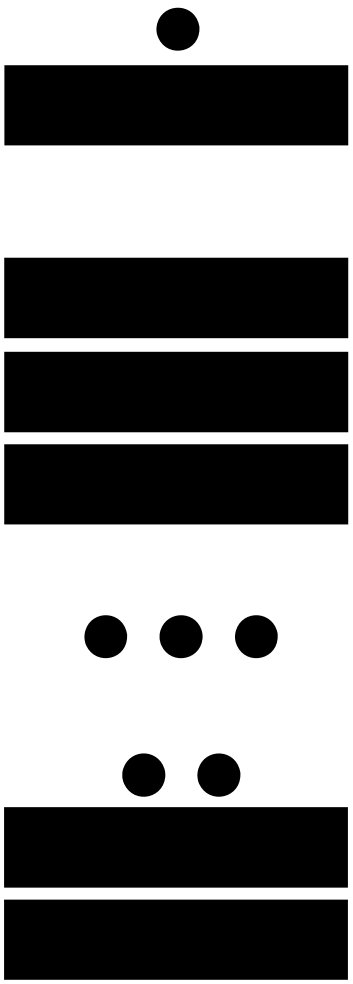
4

5

13



Larger Numbers Interpreted Using a Base 20 Place-Value System



$6 \times 20^3 = 6 \times 8{,}000 = 48{,}000$

$15 \times 20^2 = 15 \times 400 = 6{,}000$

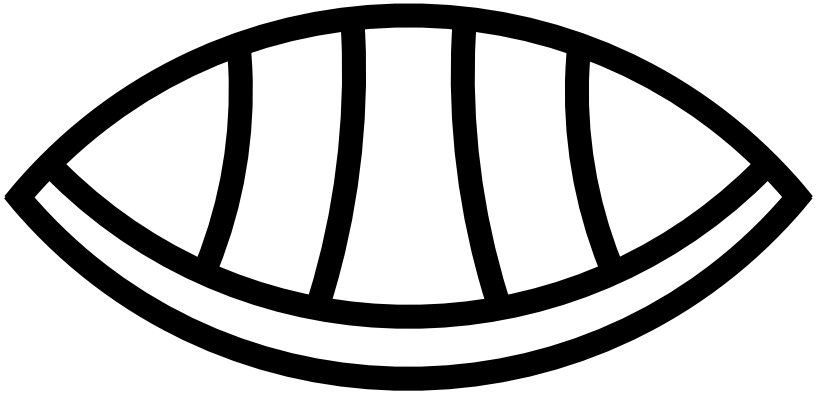
$3 \times 20^1 = 3 \times 20 = 60$

$12 \times 20^0 = 12 \times 1 = 12$

Digits are generally stacked vertically, with the units position at the bottom

Total = 54,072

Necessary for a place-value numbering system



A symbol for zero!

Perhaps the Maya did not see this as a true number; see the article “When a Number System Loses Uniqueness: The Case of the Maya—The Mayan Number System”, by Amy Shell-Gellasch and Pedro J. Freitas, in the Mathematical Association of America’s *Convergence* online resource collection, volume 9, 2012

Number words in Yucatec*, a tone language (one of 32 members of the Maya language family and often treated as the reference language for the family)

1	hun	11	buluc
2	caa, ca	12	lahca
3	ox	13	oxlahun
4	can	14	canlahun
5	hoo, ho	15	hoolahun
6	uac	16	uaclahun
7	uuc	17	uuclahun
8	uaxac	18	uaxaclahun
9	bolon	19	bolonlahun
10	lahun	20	kal, hun kal

Spellings vary by source. These are as given in Closs, p. 293. The spelling of uaxaclahun used on the next slide is from the Wikipedia article on Uaxaclajuun Ub'aah K'awiil.

*Quiché has more speakers, about 1,000,000, vs. about 770,000 for Yucatec. However, the Yucatec branch of the family originated about 1,500 years earlier than the Quiché branch.

18 Rabbits granola

The only food item I know whose brand name was translated from Yucatec (early in the era when this was first, and barely, possible)*. From Wikipedia:

Uaxaclajuun Ub'aah K'awiil (also known by the appellation "18-Rabbit" or "Eighteen Rabbit") was the 13th ajaw or ruler of the powerful Maya polity associated with the site of Copán in modern Honduras (its Classic Maya name was probably Oxwitik). He ruled from January 2, 695, to May 3, 738.

(See slide 15 for the photo credit)

*The name actually translates as "18 are the bodies of K'awiil", one of the chief Maya gods associated with divine rule. The 18 Rabbit translation dates from the early days of translation from Maya into English, when such translation was treacherous and error-prone.

Basis for Two Maya Cyclical Calendars

Three week-like blocks of days, cycling and running concurrently:

trecena (based on Spanish “trece”, 13), 13 days in each block, with days in each thought of as being numbered 1,2,3,…,13, but with first cycle starting with 4 on the Maya base date (August 11, 3144 BCE): 4,5,6,7,8,9,10,11,12,13,1,…,13,1,…,13,1,…,13,….

veintena (based on Spanish “veinte”, 20), like trecena, except 20 days in each block, but starting on the Maya base date with 20: 20,1,…,20,1,…,20,1,…,20,1,…,20,….

vague year of 365 days in each block, with days numbered 1,2,3,…,365, starting on the Maya base date with 348: 348,349,350,…,364,365,1,…,365,1,…,365,1,…,365,….

An ordered triple (trecena day, veintena day, vague year day) is associated with each day, so beginning with the base date:

(4,20,348), (5,1,349), (6,2,350), … , (8,17,365), (9,18,1), ….

Problem: This is not an absolute dating system; it is itself cyclic!

Number theory: Such ordered triples with cycling will repeat every lcm{13,20,365} = 52 x 365 = 18,980 days

Consequence: Only one-fifth of the 13 x 20 x 365 = 94,900 triplet combinations of 1,…,13, 1,…,20, 1,…,365 can appear, and (1,1,1) is not among the possibilities. CHALLENGE: Can you prove that? Spreadsheet arguments are allowed.

Today's ordered triple is (3,14,292) and was last obtained on January 30, 1970, and the next will occur on January 4, 2074.

The Maya named the veintena numbers, like we name weekdays in our 1,2,3,4,5,6,7 weekday numbering.

Veintena Day Numbers 1,2,3,…,18,19,20 Renamed

1 → Imix (water)
2 → Ik (air)
3 → Akbal (night)
4 → Kan (corn)
5 → Chicchan (serpent)
6 → Cimi (death)
7 → Manik (deer)
8 → Lamat (rabbit)
9 → Muluk (rain)
10 → Oc (dog)
11 → Chuen (monkey)
12 → Eb (broom)
13 → Ben (reed)
14 → Ix (tiger or magician)
15 → Men (bird)
16 → Cib (owl or vulture)
17 → Caban (earth)
18 → Etz'nab (flint knife)
19 → Cauac (storm)
20 → Ahau (sun god)

January 17, 2022: Its ordered triple (3,14,292) implies veintena day name Ix

Spellings and translations vary by source. These are as given in Closs, p. 331.

260-Day Maya Ceremonial Calendar: Sacred Round, or *Tzolkin* (Sequence of Days)

Elements: (trecena day, veintena day)

lcm{13,20} = 13 x 20 = 260, so length of the Sacred Round calendar is 260 days

Today's Sacred Round date = (3,14), i.e., 3 Ix; 260 days from now, on October 4, 2022 (and not until then), the Sacred Round date will again be that; (3,14), i.e., 3 Ix; but the vague year day will have changed.

What about the vague year day cycle of length 365?

365-Day Maya Civil Calendar: Vague Year, or *Haab* (just means "year")*

Broken down into 18 "months" of 20 days each, plus one extra "month" at the end of the Haab consisting of 5 days.

> "Vague" because the Maya knew that a solar year does not have exactly 365 days. There are claims that the Maya had computed the Haab to be 365.2422 days long, but many consider this claim to be unfounded, particularly since they did not do non-integer division or have a notation for fractions.

Interestingly, the ancient Egyptians had the same idea for their civil year, except that they used 12 months of 30 days each, with that extra 5-day "month" to end their year.**

*Recall that the Maya had another time interval of approximately a year, the *tun*, of 360 days, computationally useful for keeping track of the Long Count.

**And the Maya and ancient Egyptians shared (maybe not literally, despite Kon-Tiki-like conjectures) the belief that those 5 days were particularly unlucky times.

Vague Year “Month” Names

For reference, here they are:

Pop (mat)
Uo (frog)
Zip (stag)
Zotz’ (bat)
Zec (skull)
Xul (termination)
Yaxkin (red)
Mol (to gather)
Ch’en (a well)
Yax (green)
Zac (white)
Ceh (a forest)
Mac (a cover)
Kankin (yellow)
Muan (falcon or owl)
Pax (drum)
Kayab (turtle)
Cumku (granary)
Uayeb (5 unlucky days)

The spellings are from various sources that often disagree on translations; see https://thediningdiva.typepad.com/the_dining_diva/2011/04/the-maya-months.html for one that includes corresponding glyphs.

Vague Year Dates Named as One Might Expect

1 Pop, 2 Pop, 3 Pop, … 20 Pop, 1 Uo, 2 Uo, ….

January 17, 2022 has vague year day number 292, hence would be 12 Muan

A Vague Year Day as the Maya Would Represent It

oxlahun Zotz’ (13 “bat”, though the glyph does not look much like a bat), day 73 of the vague year

“Calendar Round” Dates Given as Sacred Round Date & Vague Year Date

January 27, 2022, (3,14,292), thus becomes 3 Ix 12 Muan in Maya notation.

(3, 14, 292)

trecena day 3

veintena day 14 = Ix

Tzolkin (Sacred Round) day 3 Ix

Haab (Vague Year) day 292 = 12 Muan

Calendar Round day 3 Ix 12 Muan

So far as is known, ancient texts *always* recorded Vague Year dates with the relevant Sacred Round dates conjoined at the front to form Calendar Round dates, never in isolation.

Problem

What did the classical Maya consider to be the natural human life span, and why?

Hint: If someone were born today, on 3 Ix 12 Muan, (3,14,292), how long would it be before they could celebrate their first Calendar Round birthday?

Example of a Date Year Computation from a Maya Inscription

On 8 Oc 13 Yax, (8,10,193), was born Bird-Jaguar, Lord of Yaxchilan. The distance number was then 10 kins, 5 uinals, 3 tuns, and 2 katuns (2.3.5.10), after which on 11 Ahau 8 Zec, (11,20,88), he was seated in this rulership.

Image and text reproduced from *Native American Mathematics*, Michael P. Closs, ed., University of Texas Press, pp. 310–311, specifically from Closs's article "Mathematical Notation of the Maya", pp. 291–369. The computation to follow is basically also Closs's from the succeeding pages of the article, though stylistically rewritten.

Computing One Calendar Round Date from Another, Given the Distance Number Between

At this computation's end, checking the claims made in the inscription on the preceding slide, I will share a comment Closs makes about it and a second problem that will be left as an exercise. Both are somewhat computationally involved exercises in modular arithmetic, a topic typically covered early in an undergraduate number theory course.

Start at (t_0,v_0,y_0), arriving at (t,v,y) after a distance number $n_5.n_4.n_3.n_2.n_1$

(The following is a one-time computation)

1 baktun (400 pseudoyears) = 144,000 days = -1 (mod 13), 0 (mod 20), 190 (mod 365)

1 katun (20 pseudoyears) = 7200 days = -2 (mod 13), 0 (mod 20), -100 (mod 365)

1 tun = 360 days = -4 (mod 13), 0 (mod 20), -5 (mod 365)

1 uinal = 20 days = 7 (mod 13), 0 (mod 20), 20 (mod 365)

1 kin = 1 day = 1 (mod 13), 1 (mod 20), 1 (mod 365)

Formulas

Start at (t_0,v_0,y_0), arriving at (t,v,y) after a distance number $n_5.n_4.n_3.n_2.n_1$

$$t = t_0 - n_5 - 2n_4 - 4n_3 + 7n_2 + n_1 \pmod{13}$$

$$v = v_0 + n_1 \pmod{20}$$

$$y = y_0 + 190n_5 - 100n_4 - 5n_3 + 20n_2 + n_1 \pmod{365}$$

Checking the Bird-Jaguar Computation

Bird-Jaguar was born on 8 Oc 13 Yax, represented as $(t_0,v_0,y_0) = (8,10,193)$. (Yax is the 10th month; 9 have passed.)

Distance number = 2.3.5.10 = 0.2.3.5.10

$t = 8 - 0 - 2\times2 - 4\times3 + 7\times5 + 10 = 37 = 11 \pmod{13}$

$v = 10 + 10 = 20 \pmod{20}$

$y = 193 + 190\times0 - 100\times2 - 5\times3 + 20\times5 + 10 = 88 \pmod{365}$

Ending calendar round date: (11,20,88), 11 Ahau 8 Zec, as claimed

Closs's Claim

In Closs's article, he characterizes the above computation and the more complicated one needed to solve the problem given in the next slide* (taken from Closs, pp. 315–316) as two fundamental problems in modular arithmetic the Maya knew how to solve, with the one to follow the more difficult.

Of these two, he says (pp. 307–309 of his article) that "Of the various solutions to these problems which have been proposed, the ones outlined here are probably closest in spirit to the techniques used by the ancient Maya, although the form used to express them is entirely modern."

*Skipped over in this presentation due to time constraints. skip

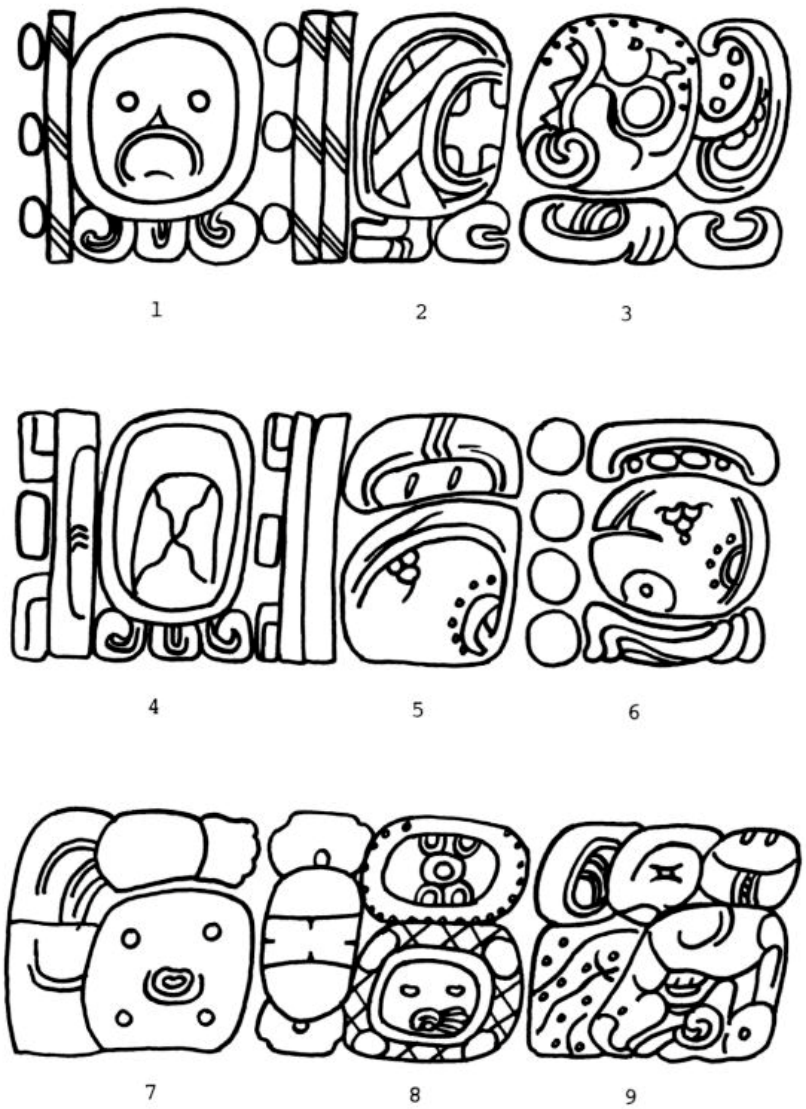
Another Problem

Pacal, perhaps the greatest of all Maya kings, had this inscription on his sarcophagus:

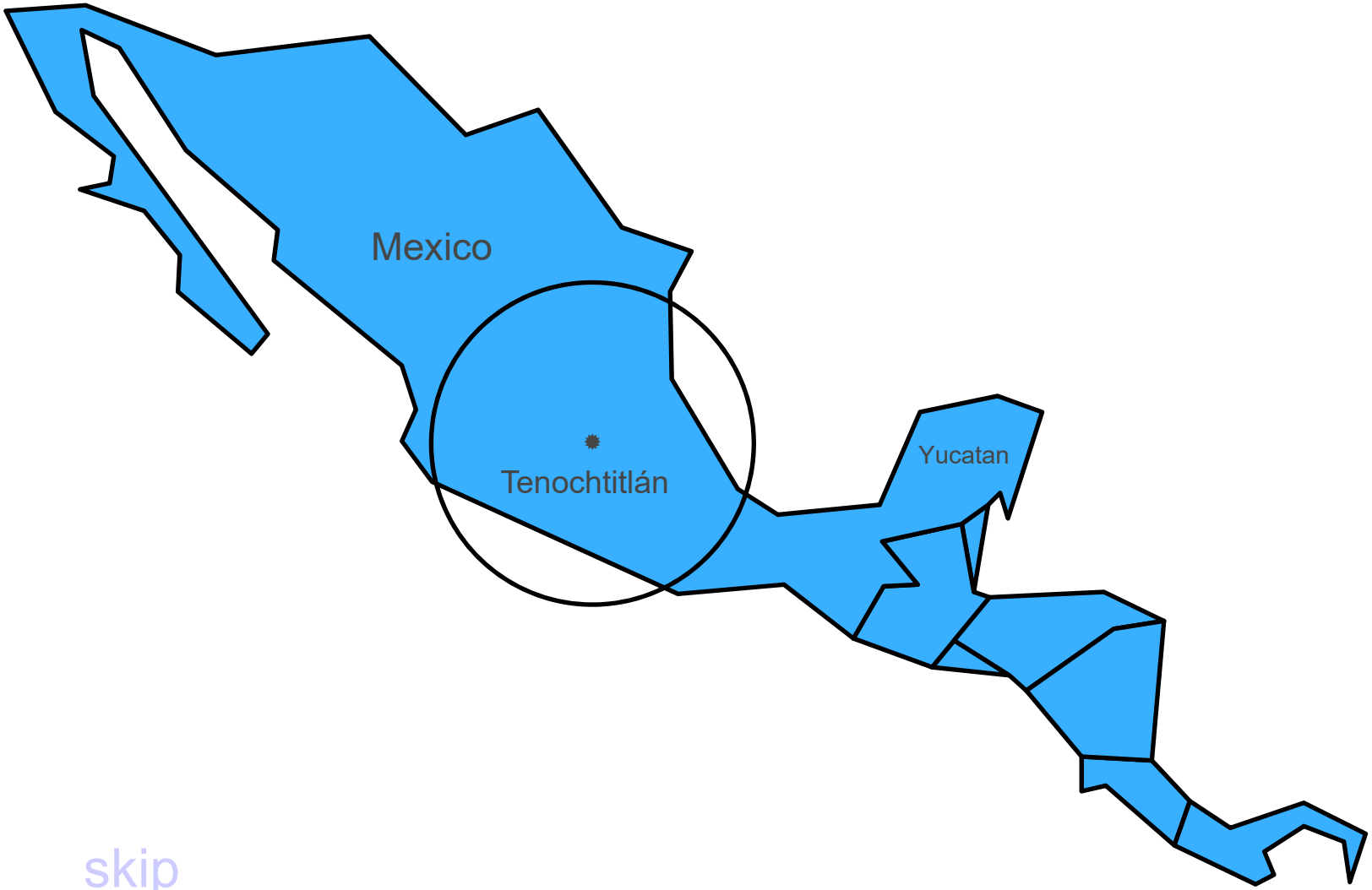
> On 8 Ahau 13 Pop he was born; on 6 Etz'nab 11 Yax he died, having lived through parts of four katuns; King Pacal, Lord of Palenque.

Problem: How old was Pacal when he died?

HINT: The part about the four katuns places his age between 60 and 100.



Homelands of the Mexica (Aztec)



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Calendrical System: Clearly borrowed from the Maya

Sacred Cycle: Number (1–13) + Day Name (cycle of 20) gives cycle of 260 sacred day designations

Vague year of eighteen named 20-day months plus an additional 5 day period

Special feature: Vague year known by the sacred day name of its 360th day

- Of the 260 possible sacred day names, 52 (one fifth!) are used
- Cycle: One person's natural lifetime

Legend of Ce Acatl Topiltzin (1 Reed the Divine), sometimes conflated with the Mexica deity Quetzalcoátl

skip

This is disputed!

Legendary ruler of the Toltecs ("old ones"), builders of Teotihuacan, precursors of the Mexica

Considered to be a white god

Legend: Ce Acatl Topiltzin would return to resume his rule in a year named Ce Acatl (1 Reed)

(Acatl: 13th sacred day name, equivalent of the Maya's Ben, which *also* means Reed.)

Accompanied by only 508 soldiers, about 100 sailors, 16 horses, and minimal quantities of firearms and artillery,

Cortés arrived in 1519, in the year Ce Acatl!

Though the Mexica empire was mighty, with at least 400,000 warriors, this paralyzed Moctezuma II, the emperor, and the rest is history.

Extended presentation, including everything skipped, is posted in its entirety at:
http://www-personal.umich.edu/~meggin/IndigenousMath.pdf

A final note and exercise for you on the quirkiness of even more modern calendrical systems

A research problem: George Washington was born February 11, 1731 Julian, converted later to February 22, 1732 Gregorian

Since Pope Gregory's calendar revision moved the date only a few days forward, why the difference in year?