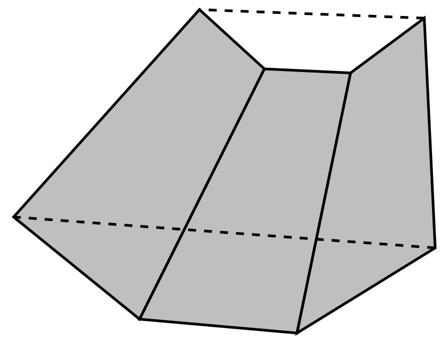
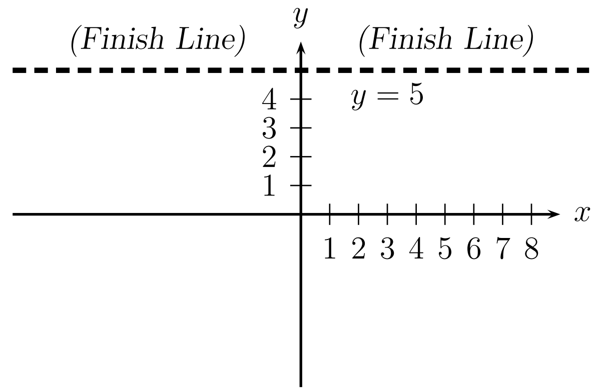
Douglass Houghton Workshop, Section 2, Thu 04/09/20

Worksheet To Infinity, and Beyond



1. The picture to the right shows a section of the Los Angeles river, whose sides are lined with concrete. It is currently full of water, but we need to empty it so we can film a car chase scene for a movie (as in *Terminator 2*, *Grease*, *Gone in 60 Seconds*, *Buckaroo Banzai*, etc.) It is 100 meters long, 17 meters deep, 40 meters wide at the top and 20 meters wide at the bottom. Find the work required to pump all the water up to the top of the river.

The Battlefield



2. (This problem appeared on a Winter, 2003 Math 116 exam) The newest FOX reality show, "BattleBugs: Clash of the Beetles" begins (at $t = 0$) with eight assorted insects placed randomly on a large mat (the "battlefield", pictured here), on which is marked a "finish line". The producers hoped that the bugs would battle to be first to cross the finish line, but instead they wander around, each according to its nature. The motion of each bug is described by the equations below. Both x and y are measured in inches.

Hercules Beetle $x(t) = \cos(t/2)$ $y(t) = \sin(t/2)$	**Ladybug** $x(t) = e^{-t}$ $y(t) = e^{-2t}$	**Tiger Beetle** $x(t) = 1 + t$ $y(t) = -1 + 8t$	**Longhorned Beetle** $x(t) = 3 + t$ $y(t) = 4 - t$
Dung Beetle $x(t) = t$ $y(t) = -2$	**Scarab** $x(t) = 2 - 7t$ $y(t) = -1 - 7t$	**June Beetle** $x(t) = 0$ $y(t) = -1$	**African Ground Beetle** $x(t) = \sin(t)$ $y(t) = \cos(t)$

Which bug (or bugs)...

(a) move repetitively?

(b) begin closest to the finish line?

(c) move fastest?

(d) will move very slowly (or not at all), in the long run?

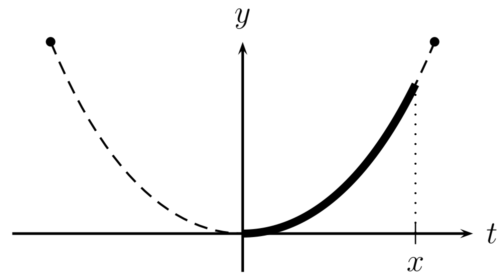
(e) will reach the finish line first?

(f) gets the dizziest?

3. We've made some progress finding the shape of a hanging chain. If the shape is given by $F(x)$, then by considering forces and arc length we've shown that

$$T_0 F'(x) = \delta g \int_0^x \sqrt{1 + F'(t)^2}\, dt$$

where T_0 is the tension at the bottom of the chain, δ is the mass density of the chain, and g is acceleration due to gravity (all constants). Where to go from here? We'd like to find a formula for $F(x)$.



(a) That thing on the right is begging for you take its derivative. ("Take my derivative!" it cries.) So take the derivative of both sides with respect to x.

(b) Hmmm. No Fs, only F's. And lots of constants. Let $y = F'(x)$, and put all the constants together into one constant. That should make it look better.

(c) What is y when x is 0? Now you have an initial value to go with your differential equation.

(d) Separate the variables and solve the differential equation.

4. (From the Fall, 2018 Math 116 Final Exam) Consider the curve $y = \sqrt{1 - x^2}$. Suppose a paperweight is formed by rotating this curve around the x-axis. This paperweight has a density given by $\delta(x) = 2 + \cos(x)\ \mathrm{g/cm}^3$. The units on both axes are centimeters (cm).

(a) Write an expression that gives the approximate mass, in grams, of a slice of the paperweight taken perpendicular to the x-axis at coordinate x with thickness Δx. (Assume that Δx is small but positive.) Your expression should <u>not</u> involve any integrals.

(b) Write an expression involving one or more integrals that gives the mass, in grams, of the paperweight.