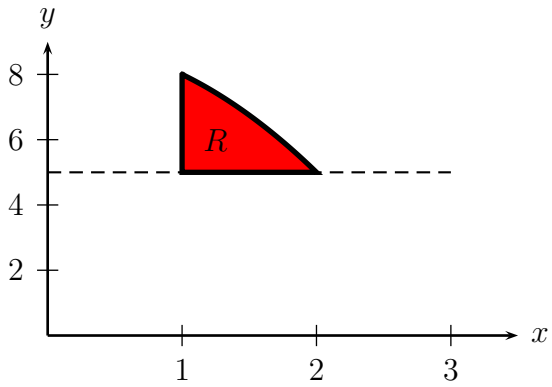
Douglass Houghton Workshop, Section 2, Thu 03/26/20

Worksheet Poseidon

1. (Adapted from a Fall, 2011 Math 116 Exam) Consider the region R in the xy-plane bounded by the curves $y = 9 - x^2$, $x = 1$, and $y = 5$. This region is pictured below.



   (a) Find the area of R.

   (b) Find the volume obtained by rotating R about the y-axis. Do it with both shells and washers, and verify that the answer is the same.

   (c) Find the volume of the solid whose base is R and whose cross-sections perpendicular to the x-axis are squares.

2. It's an interesting idea to start with a sequence of numbers $a_0, a_1, a_2, \ldots$ and try to find a formula for the function with Taylor series $a_0 + a_1x + a_2x^2 + \cdots$. Consider the Fibonacci numbers:

n	0	1	2	3	4	5	6	7	8	9
F_n	0	1	1	2	3	5	8	13	21	34

   where, for $n \geq 2$, $F_n = F_{n-1} + F_{n-2}$.

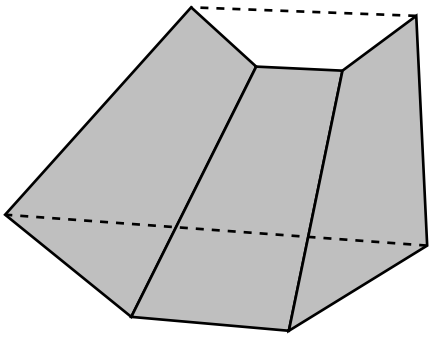
   Suppose $f(x) = F_0 + F_1x + F_2x^2 + \cdots$. (It's called the *generating function* for the Fibonacci numbers.)

   (a) Write down the first 10 terms of the series for $f(x)$ and $xf(x)$.

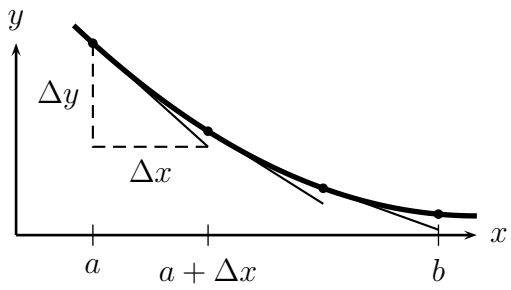
   (b) What happens when you add those two together? Compare with $f(x)/x$.

   (c) Deduce a simple formula for $f(x)$.

3. The picture to the right shows a section of the Los Angeles river, whose sides are lined with concrete. It is currently full of water, but we need to empty it so we can film a car chase scene for a movie (as in *Terminator 2*, *Grease*, *Gone in 60 Seconds*, *Buckaroo Banzai*, etc.) It is 100 meters long, 17 meters deep, 40 meters wide at the top and 20 meters wide at the bottom. Find the work required to pump all the water up to the top of the river.



4. How can we compute the length of a curve $y = f(x)$? Consider cutting it up into small pieces, and approximating each piece with a line segment, as in the picture below.



   (a) How long is the first piece? It is tangent to the curve at a.

   (b) How long is the ith piece?

   (c) Write the left-hand Riemann sum for the length of the curve from a to b.

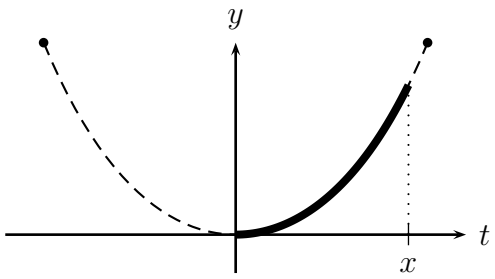
   (d) Now make it into an integral, which will be our formula for arc length.

5. Last time we enumerated the forces on the piece of hanging chain shown here: gravity (mg), leftward tension (T_0) and tension pulling up and to the right (T).

   (a) T can be split into horizontal and vertical components. What are their sizes? (Remember the forces must sum to 0.)

   (b) Let $y = F(t)$ be the shape of the chain. What does the fact that T is pulling in the direction of the chain tell you about the slope of F?

   (c) T_0 is constant as x changes. But the force of gravity is not. Why? How could we find the weight of the piece of chain if we knew $F(t)$?



6. You can describe Secret Santa games in cycle notation like this: (142)(35) means person 1 gives to person 4, 4 gives to 2, and 2 gives to 1; 3 gives to 5 and 5 gives to 3. Note that (142), (421), and (214) all represent the same cycle, so to avoid confusion, we always write cycles with the lowest number first.

   A game is successful if everyone gives to someone else, i.e., no one is in a cycle by themselves. Write down all the successful games for 2, 3, and 4 people. Make a table with the number of successful games for $n = 1, 2, 3, 4$ people.

7. Suppose you are pumping water up from a lake to a water tank. The tank is a rectangular solid, with a base that is $46'' \times 38\frac{1}{2}''$, and a height of $38''$. The base is 27 feet above the lake. Water weighs $62.4\,\text{lb/ft}^3$.

   (a) How much work, in ft·lb, will it be to fill the tank?

   (b) It took about 10 oz of gasoline to pump the water up. A gallon of gasoline contains about 132 megajoules of energy, according to Wikipedia. Use the fact that 1 gallon is 128 ounces and 1 ft·lb is 1.355 joules to find the efficiency of the pump.