Douglass Houghton Workshop, Section 1, Mon 03/16/20

Worksheet Lumberjacks are OK

1. Let

$$\begin{aligned} S &= \text{The fraction of people who are susceptible to infection,} \\ I &= \text{The fraction of people currently infected, and} \\ R &= \text{The fraction of people who have been removed.} \end{aligned}$$

   Suppose also that the probability that the disease is transmitted during a single interaction between an infected and susceptible person is .01 (1%).

   (a) Suppose Bob is a susceptible person, and on a given day he interacts with n other people, chosen randomly. About how many infected people did he interact with?

   (b) What's the probability Bob was NOT infected the first time he met an infected person AND not infected the second time he met an infected person? Assume the interactions are independent.

   (c) What's the probability that Bob wasn't infected by any of his interactions?

   (d) What's the last answer if the probbility of transmission is p instead of .01?

2. So the probability that Bob becomes infected is

$$1 - (1-p)^{nI}$$

   where p is 1% or less, and nI is on the order of 10 or 20.

   (a) Find the first two nonzero terms of the Taylor series about $x = 0$ for

$$f(x) = (1-x)^k$$

   where k is some constant.

   (b) Use that to approximate the probability that Bob becomes infected.

3. So we can model the disease's spread by saying that the three types of people change accoring to these equations:

$$\begin{aligned} S(t+1) &= S(t) - npI(t)S(t) \\ I(t+1) &= I(t) + npI(t)S(t) - vI(t) \\ R(t+1) &= R(t) + vI(t) \end{aligned}$$

   where n is the number of contacts per person, p is the probability of one contact between an S and an I resulting in infection, and v is the removal rate.

   Use the Google sheet for your group to model the disease using those equations. n, p, and v are defined to refer to the numbers in the upper-right corner of the sheet. So you can use those letters in your equations. Once you get it to work, try changing the value of n to see what happens.