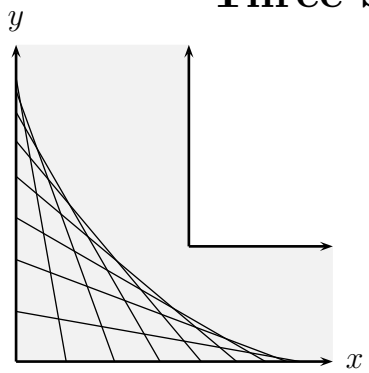
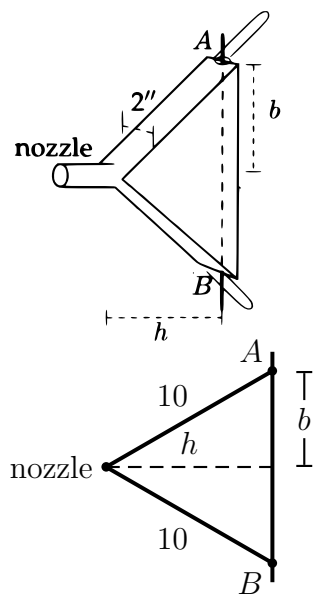
Three Sample DHSP Problems



1. Suppose we are carrying a ladder down a hallway, and then turning it to get around a corner, always keeping the ends of the ladder against the walls. The question is: **Which points on the floor does the ladder pass over?** The thin lines on the picture represent the ladder, and the thick lines are the hallway. Find the curve that bounds all the thin lines.

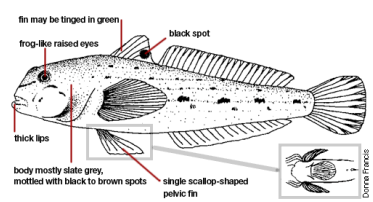
2. (This problem appeared on a Math 115 exam.) A bellows has a triangluar frame made of three rigid pieces. Two pieces, each 10 inches long, are hinged at the nozzle. They are attached to the third piece at points A and B which can slide, as shown in the diagrams below. (The figures show a 3D sketch of the bellows and a 2D sketch that may be specifically useful to solve the problem.)
Each piece of the frame is 2 inches wide, so the volume (in cubic inches) of air inside the bellows is equal to the area (in square inches) of the triangluar cross-section above, times 2. Suppose you pump the bellows by moving A downward toward the center at a constant speed of 3 in/sec. (So B moves upwards at the same speed.) What is the rate at which air is being pumped out when A and B are 12 inches apart? (So A is 6 inches from the center of the vertical piece of the frame.)



3. You are studying the population of Round Gobies, an invasive fish species, in Lake Michigan. Suppose that the population changes according to the rule:

$$P(n+1) = 1.5P(n) - 200$$

where $P(0)$ is the population in 2015, $P(1)$ is the population 1 year later, etc.



(a) Make up a (short) story about Round Gobies that explains the formula.
(b) Suppose there are 320 fish in 2015. What will happen in the long run?
(c) Suppose instead that there are 800 fish in 2015. Now what happens?
(d) A population is in **equilibrium** if it stays the same from year to year. Is there an equilibrium number for this population?
(e) We could do this same problem for any rule of the form $P(n+1) = mP(n) + b$. How would changing m and b affect the outcome?