

Proof of angle sum formulas

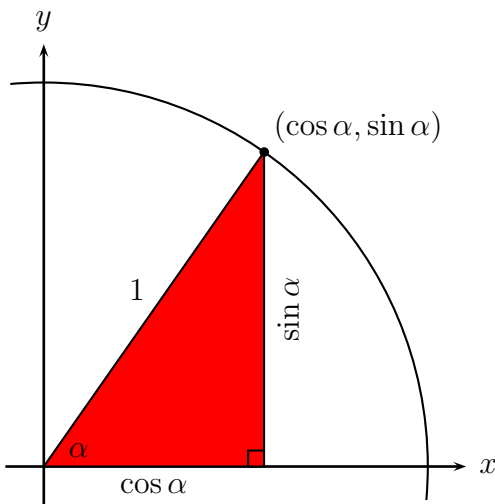
Julian asked me for a geometric proof of the angle sum formulas:

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$$

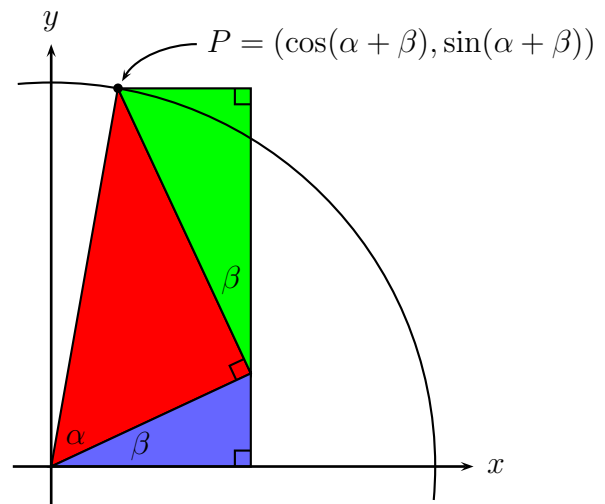
$$\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$$

The proof below will hopefully convince you that the formulas work when α and β are acute angles (i.e. $0 < \alpha, \beta < \frac{\pi}{2}$).

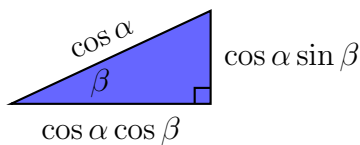
First, draw the angle α the way we normally draw it on the unit circle.



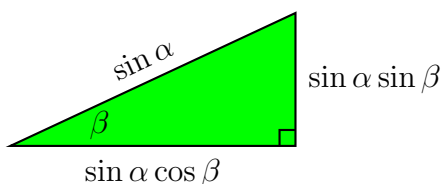
Then rotate the red triangle counterclockwise by the angle β , and draw horizontal and vertical lines to make two new triangles. Note the coordinates of P .



The blue triangle has a vertex angle β and a hypotenuse of length $\cos \alpha$. So we can fill in the lengths of the other sides:



Likewise the green one:



Now we can calculate the coordinates of P using the edges of the blue and green triangles, and obtain the identities.

