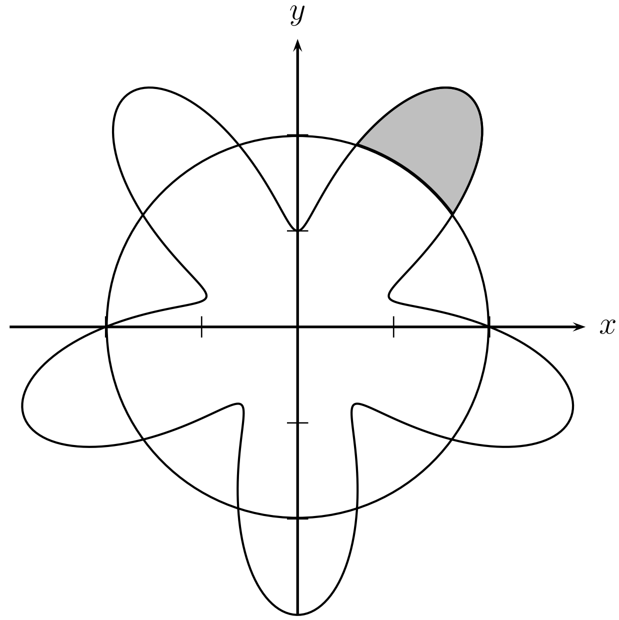
Douglass Houghton Workshop, Section 2, Tue 04/14/20

Worksheet Until We Meet Again

1. (Adapted from a Fall, 2010 Math 116 Exam) In the picture to the right, the graphs of $r = 2$ and $r = 2 - \sin(5\theta)$ are shown.



   (a) Write a definite integral that computes the shaded area.

   (b) Compute the area exactly.

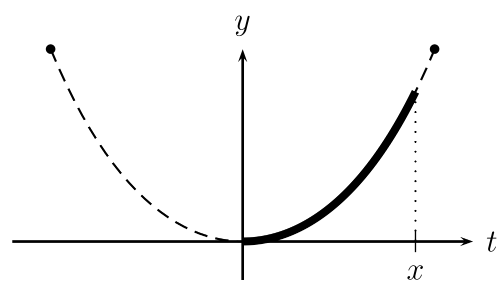
   (c) Write an integral for the length of the boundary of the shaded area.

   (d) Get an approximate answer for that length, using your calculator.

2. We've made some progress finding the shape of a hanging chain. If the shape is given by $F(x)$, then by considering forces and arc length we've shown that

$$T_0 F'(x) = \delta g \int_0^x \sqrt{1 + F'(t)^2}\, dt$$

where T_0 is the tension at the bottom of the chain, δ is the mass density of the chain, and g is acceleration due to gravity (all constants). Where to go from here? We'd like to find a formula for $F(x)$.



   (a) That thing on the right is begging for you take its derivative. ("Take my derivative!" it cries.) So take the derivative of both sides with respect to x.

   (b) Hmmm. No Fs, only F's. And lots of constants. Let $y = F'(x)$, and put all the constants together into one constant. That should make it look better.

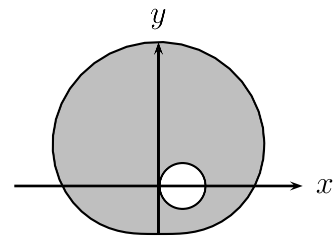
   (c) What is y when x is 0? Now you have an initial value to go with your differential equation.

   (d) Separate the variables and solve the differential equation.

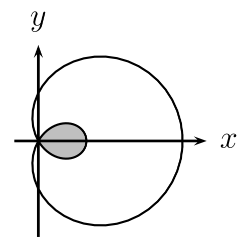
3. (This problem is from a Fall, 2014 Math 116 exam. For some reason, all the exams that term were about robots and chickens.)

Consider the polar curves

$$r = \cos\theta \qquad \text{and} \qquad r = \sin\theta + 2.$$



(a) Franklin's robot army occupies the shaded region between these two curves. Find the area occupied by Franklin's robot army.



(b) Your friend, Kazilla, pours her magic potion on the ground. Suddenly, a flock of wild chickens surrounds you. The chickens occupy the shaded region enclosed within the polar curve $r = 1 + 2\cos\theta$ as shown below. Find an integral for the perimeter of the region occupied by the flock of wild chickens.