Douglass Houghton Workshop, Section 2, Thu 04/02/20

Worksheet Rendezvous with Destiny

1. On April first, Lexi and Katie like to play practical jokes on people. Their rule, of course, is that every practical joke done on them demands an equal and opposite retaliation. From year to year they keep a mental "balance sheet" that records how much grief they "owe" or are "owed" by their rivals. Due to various interventions by authority, they have been very good the last few years, and they currently "owe" 100 practical jokes.

   They decide that every year, they will pay off 20% of their "debt", by playing pranks on their friends. At the same time, their friends play 5 pranks on them each year.

   (a) What will the "balance" be at the end of 4/1/2020?

   (b) Fill in the table with the balance B_n at the end of $4/1/(2019+n)$.

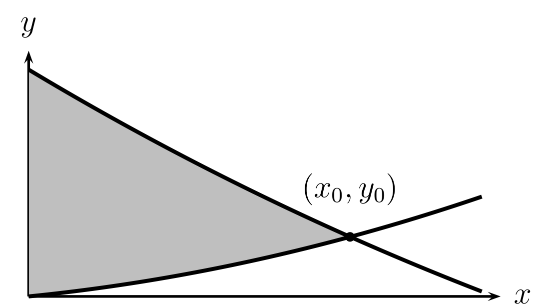
n	0	1 (2020)	2 (2021)	3	4	5
B_n	100					

   (c) Find a formula for B_n in terms of n.

   (d) What happens in the long run? (Does the sequence $B_0, B_1, B_2, \ldots$ converge?)

2. (Fall, 2007) Find the interval of convergence of $\displaystyle\sum_{n=3}^{\infty} \frac{(3-x)^{3n}}{8^n(n-2)}$.

3. (From the Fall, 2010 Math 116 final) The graph shows the area between the graphs of $f(x) = 6\cos(\sqrt{2x})$ and $g(x) = x^2 + x$. Let (x_0, y_0) be the intersection point between the graphs of $f(x)$ and $g(x)$.



   (a) Compute $P(x)$, the function containing the first three nonzero terms of the Taylor series about $x = 0$ of $f(x) = 6\cos\left(\sqrt{2x}\right)$.

   (b) Use $P(x)$ to approximate the value of x_0.

   (c) Use $P(x)$ and the value of x_0 you computed in the previous question to write an integral that approximates the value of the shaded area. Find the value of this integral.

   (d) Graph $f(x)$ and $g(x)$ in your calculator. Use the graphs to find an approximate value for x_0.

   (e) Write a definite integral in terms of $f(x)$ and $g(x)$ that represents the value of the shaded area. Find its value using your calculator.

4. Determine whether the following improper integrals converge or diverge.

(a) $\displaystyle\int_1^\infty \frac{1}{x+e^x}\,dx$ (b) $\displaystyle\int_1^e \frac{1}{x(\ln(x))^2}\,dx$

5. (This problem appeared on a Fall, 2004 Math 116 exam.)

(a) Find the second order Taylor polynomial of $f(x) = \sqrt{4+x}$ for x near 0.

(b) Find the Taylor series about $x = 0$ of $\sin(2x)$, either from scratch or by using a series you know already.

(c) Using your answers to parts (a) and (b) and *without computing any derivatives*, find the second order Taylor polynomial that approximates $g(x) = \sqrt{4+\sin(2x)}$ for x near 0.

(d) The error in a Taylor polynomial approximation is mostly in the first term omitted, in this case the third order term. So compute that, and give an estimate of the maximum error in the approximation when $-0.1 \le x \le 0.1$.