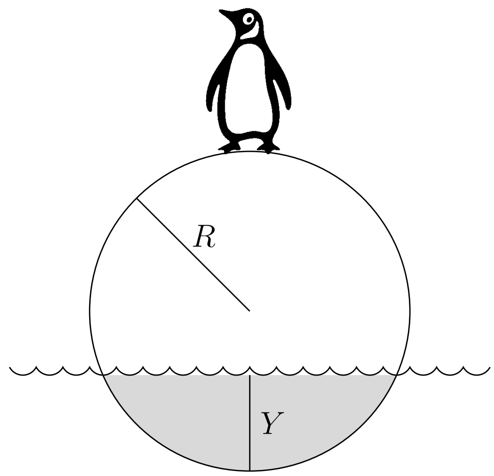
Douglass Houghton Workshop, Section 2, Tue 03/24/20

Worksheet One Ring to Rule Them All

1. The buoyancy force on a floating object is proportional to the volume of water it displaces. Using this fact, whimsical ecologists plan to study the weights of penguins by floating beachballs on the ocean and enticing penguins to climb on top. They then measure the depth the ball sinks to, and thereby deduce the penguin's weight.

   So given a beachball of radius R that is partially submerged in the water, find a formula for the volume of the ball which is below the water line when its bottom is at depth Y. Check that your formula makes sense for the values $Y = 0$, $Y = R$, and $Y = 2R$.

2. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0. Gravity causes the ball to accelerate downward at a constant rate, g. (This might be on another planet, so use g rather than $9.8\,\mathrm{m/sec}^2$.)

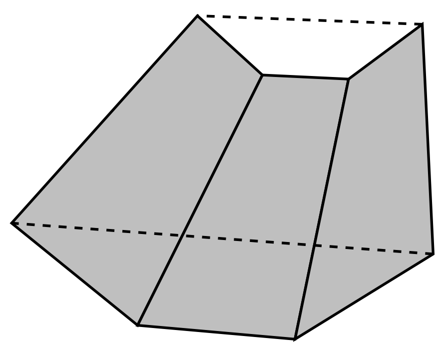
   (a) Find $v(t)$, the upward velocity of the ball at time t.

   (b) Find $h(t)$, the height of the ball at time t.

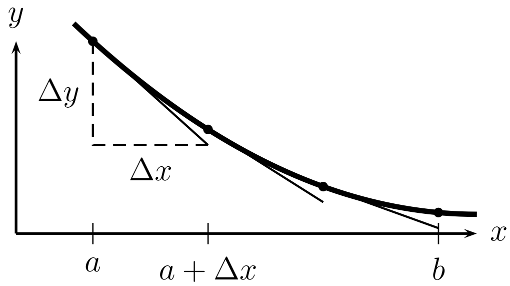
   (c) Calculate the quantity $mgh(t)+\frac{1}{2}mv(t)^2$. What do you notice about your answer?

   (d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.

3. The picture to the right shows a section of the Los Angeles river, whose sides are lined with concrete. It is currently full of water, but we need to empty it so we can film a car chase scene for a movie (as in *Terminator 2*, *Grease*, *Gone in 60 Seconds*, *Buckaroo Banzai*, etc.) It is 100 meters long, 17 meters deep, 40 meters wide at the top and 20 meters wide at the bottom. Find the work required to pump all the water up to the top of the river.

4. How can we compute the length of a curve $y = f(x)$? Consider cutting it up into small pieces, and approximating each piece with a line segment, as in the picture below.



(a) How long is the first piece? It is tangent to the curve at a.

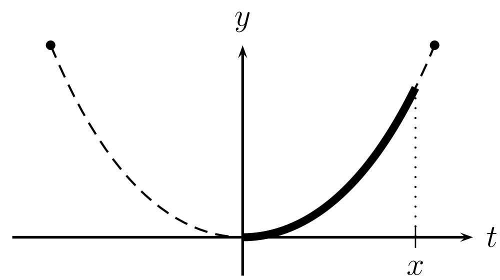
(b) How long is the ith piece?

(c) Write the left-hand Riemann sum for the length of the curve from a to b.

(d) Now make it into an integral, which will be our formula for arc length.

5. In our quest to determine the shape of a hanging chain, we have found that the forces on a portion of the chain obey a certain relationship: if $m(x)$ is the mass of the chain between the middle and position x, T_0 is the tension in the chain at the bottom, and $y = F(x)$ is the shape of the chain, then in order to make the forces balance we must have:

$$\frac{m(x)g}{T_0} = F'(x).$$



(a) How could you calculate $m(x)$ if you knew $F(x)$?

(b) Some of that we know how to do. Use it to modify the equation above. Feel free to combine unknown constants into one, and simplify as much as possible.