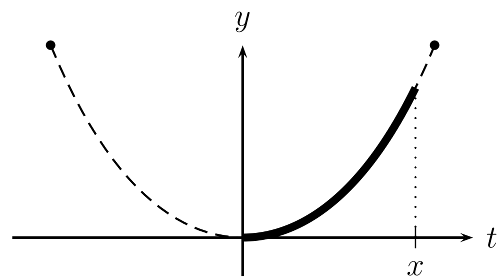
Douglass Houghton Workshop, Section 2, Tue 03/10/20

Worksheet Love looks not with the eyes, but with the mind

And therefore is winged Cupid painted blind



1. We're interested in finding an equation that describes the shape of a hanging chain. Clearly the shape is determined by the forces on the chain.
   (a) Consider the portion of the chain highlighted here. Draw it on the board, and draw arrows for all the forces that act on it.
   (b) Give the forces names. Given that the chain is not in motion, what must the forces sum to?
   (c) So how are your variables related? Write down as many equations as you can.
2. (From the Fall, 2013 Math 116 Final Exam) Find the interval of convergence of
$$\sum_{n=1}^{\infty} \frac{2^n}{3n}(x-5)^n.$$
   and rigorously justify you answer, explaining what convergence test(s) you use and how you used them.
3. Write down the Taylor series about $a = 0$ for the following functions, either from memory or by working them out.

   (a) $e^x =$ (c) $\cos(x) =$ (e) $\cosh(x) = \frac{1}{2}(e^x + e^{-x}) =$

   (b) $e^{-x} =$ (d) $\sin(x) =$ (f) $\sinh(x) = \frac{1}{2}(e^x - e^{-x}) =$
4. We know by the integral test that $\zeta(2) = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{n^2} + \cdots$ converges. But what does it converge to?
   (a) Use your calculator to find the first dozen or so partial sums. Can you guess what the limit is? If you like, type in the calculator program on the right and let it run, to see how the partial sums change.

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0 → S
1 → N
Lbl 10
S+1/N² → S
N+1 → N
Disp S
Goto 10
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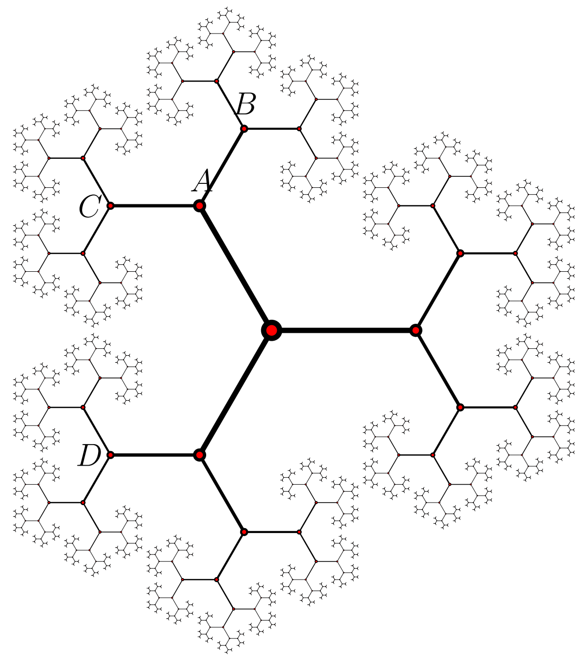
   (b) Before Spring Break we found the Fourier Series for x^2, namely
$$x^2 = \frac{\pi^2}{3} + \sum_{n=1}^{\infty}(-1)^n \frac{4}{n^2}\cos(nx).$$
   It wasn't clear why we wanted to do that—it seemed silly to write x^2 in terms of an infinite sum of cosines. But now: plug in $x = \pi$ and see if you can find $\zeta(2)$.

5. The figure below contains only 120° angles. As you move away from the center, the line segments get shorter by a factor r. That is, the longest segments (connected to the center) have length 1, the next longest have length r, the next longest after that have length r^2, etc. Assume the branches keep splitting and splitting, ad infinitum. Most of your answers will be in terms of r, but we'll be able to find what r is in part (h). No '. . .' or '$\sum$' allowed in any of your answers.



   (a) Suppose you start at the center and follow the generally northward path. That is, go to A, then turn right and go to B, then turn left, right, left, etc. How far will you travel after n steps? How far will you travel if you take an infnite number of steps?

   (b) If you take the path described in part (a), how far to the north will you have gone when you reach A? (That is, how much higher on the page is A than the center?) How far north will you have gone when you reach B? When you have gone n steps?

   (c) Use the result of part (b) to give the total height of the figure.

   (d) Find the distance from the center to the left side of the figure by following the generally northwestward path that goes to A, then turns left to C, then right, left, right, etc. This time you want the horizontal distance travelled.

   (e) Find the distance from the center to the right side of the figure by following a generally northeastward path. Thus find the total width of the figure.

Now of course, the picture could be drawn with any value of r. But if r were too large, the figure would overlap itself, and if r were too small, it would hard to see what's going on. The picture above was drawn by using the largest possible value of r which doesn't cause overlap. Thus the path that goes generally southward from C never crosses the path that goes generally northward from D, but they do approach the same point.

   (f) Find the vertical distance from C to D by using a path through the center.

   (g) Find the same distance by considering the southward path from C and the northward path from D.

   (h) Set them equal and solve for r. Do you recognize this number?