Douglass Houghton Workshop, Section 2, Tue 02/18/20

Worksheet Hamster

1. Find the probability of winning the pass bet in craps.

2. Let's prove that a series converges.

   (a) Draw the graph of $y = 1/x^2$ for $x > 0$.

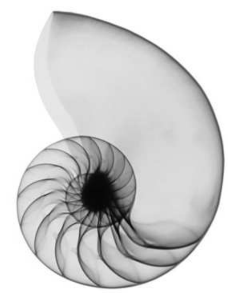
   (b) Draw some rectangles representing the Right Hand Sum with $\Delta x = 1$ for $\int_1^\infty dx/x^2$.

   (c) Explain why this proves that $\sum_{n=2}^{\infty} 1/n^2$ converges.

   (d) Of course $\sum_{n=1}^{\infty} 1/n^2$ is just one bigger than that last sum, so it converges too. But what does it converge to?

3. Use an argument similar to the last one to prove that $\sum_{n=1}^{\infty} 1/n$ diverges.

4. (6 pts) (This problem appeared on a Winter, 2003 Math 116 exam) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. Show your work on both parts.



   (a) How much volume is enclosed by the last 15 chambers constructed?

   (b) How much volume is enclosed by *all* the chambers? Assume for simplicity that there are infinitely many chambers.

5. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

$$\int_1^\infty \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

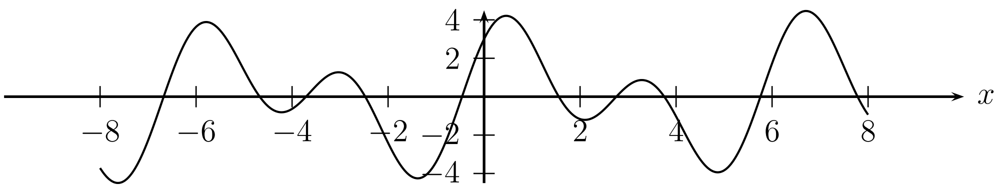
differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.

6. Determine whether the following improper integrals converge or diverge.

   (a) $\displaystyle\int_1^\infty \frac{1}{x+e^x}\,dx$ (b) $\displaystyle\int_1^e \frac{1}{x(\ln(x))^2}\,dx$

With a lot of hard work, we filled the table to the right with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column, and m and n are positive integers.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$

7. 

Suppose $h(x)$ is some function you measure in nature, and its graph looks like the one above. You do some numerical integration and discover that

$$\int_{-\pi}^{\pi} h(x)\,dx = 0 \qquad \int_{-\pi}^{\pi} h(x)\cos(2x)\,dx = 6.28$$

$$\int_{-\pi}^{\pi} h(x)\cos(x)\,dx = 3.14 \qquad \int_{-\pi}^{\pi} h(x)\sin(2x)\,dx = 4.71$$

$$\int_{-\pi}^{\pi} h(x)\sin(x)\,dx = 6.28 \qquad \int_{-\pi}^{\pi} h(x)\cos(nx)\,dx = \int_{-\pi}^{\pi} h(x)\sin(nx)\,dx = 0 \text{ for } n \geq 3.$$

Can you guess a formula for $h(x)$? Use what you know, and check by graphing your formula on a calculator.

8. We learned recently that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

as long as $|x| < 1$. You can think of both sides of the equation as *functions of* x, and so we have the suprising new idea that a common function we're familar with, $\frac{1}{1-x}$, can be expressed as an infinite series, for certain values of x.

Maybe other functions we know and like can also be expressed as series? Suppose that:

$$e^x = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$$

for some constants $a_0, a_1, a_2, \ldots$, for all x.

(a) What must a_0 be? Hint: plug in 0 to both sides.

(b) Take the derivative of both sides. Now deduce a_1.

(c) Repeat to find $a_2, a_3, a_4, \ldots$.

(d) Can it really be true?!? Try to test with the first 10 terms of the series and $x = 1$.