Douglass Houghton Workshop, Section 2, Thu 02/13/20

Worksheet Giraffe

1. Consider the "Puppy Paws" bet in craps. The bet wins on double fives (⚄⚄) and loses on "soft ten" (⚃⚅ or ⚅⚃) and on 7. If something other than a 7 or 10 is rolled, the bet stays through the next roll.

   (a) Draw the addition table below on the board and fill it in.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

   (b) Calculate these probabilities:
   - $W =$ the probability of winning on the first roll.
   - $L =$ the probability of losing on the first roll.
   - C = the probability that the game continues to a second roll.

   (c) Calculate the probability of winning on the *second* roll.

   (d) Calculate the probability of winning on the kth roll.

   (e) Calculate the probability of winning on *one of* the first n rolls.

   (f) Calculate the probability of winning the Puppy Paws bet.

2. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

$$\int_1^\infty \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

   roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

   differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.

3. Determine whether the following improper integrals converge or diverge.

   (a) $\displaystyle\int_1^\infty \frac{1}{x+e^x}\,dx$ (b) $\displaystyle\int_1^e \frac{1}{x(\ln(x))^2}\,dx$

We've done a lot of integrals with sines and cosines, and filled in this table with the results of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$:

$f \setminus g$	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	π if $m = n$, 0 if $m \neq n$	0
$\cos(mx)$	0	0	π if $m = n$, 0 if $m \neq n$

4. Let $h(x) = 5 + 2\cos(x) + \sin(x) - 5\cos(2x) + 3\sin(2x)$.

   (a) Use your calculator to compute:

   $$\int_{-\pi}^{\pi} h(x)\,dx =$$

   $$\int_{-\pi}^{\pi} h(x)\cos(x)\,dx =$$

   $$\int_{-\pi}^{\pi} h(x)\sin(x)\,dx =$$

   $$\int_{-\pi}^{\pi} h(x)\cos(2x)\,dx =$$

   $$\int_{-\pi}^{\pi} h(x)\sin(2x)\,dx =$$

   (b) Explain the results using the table above.

5. Predict what the integrals in (4a) above will be if we change $h(x)$ to

   $$h(x) = 2 + 3\cos(x) - 7\sin(x) - 4\cos(2x) + \sin(2x).$$

6. Generalize: What will those integrals be if

   $$h(x) = a_0 + a_1\cos(x) + b_1\sin(x) + a_2\cos(2x) + b_2\sin(2x).$$

7. (6 pts) (This problem appeared on a Winter, 2003 Math 116 exam) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. Show your work on both parts.

   (a) How much volume is enclosed by the last 15 chambers constructed?

   (b) How much volume is enclosed by *all* the chambers? Assume for simplicity that there are infinitely many chambers.

8. Prove whether the following improper integrals converge or diverge.

   (a) $\displaystyle\int_3^{\infty} \frac{\ln(x)}{x^2}\,dx$ (b) $\displaystyle\int_0^{\infty} \frac{3}{4x^2 + 5\sqrt{x}}\,dx$