Douglass Houghton Workshop, Section 2, Thu 01/30/20

Worksheet Fate, or Folly?

1. Recall that:

$$\begin{aligned}
\sin(x+y) &= \sin(x)\cos(y) + \cos(x)\sin(y)\\
\sin(x-y) &= \sin(x)\cos(y) - \cos(x)\sin(y)\\
\cos(x+y) &= \cos(x)\cos(y) - \sin(x)\sin(y)\\
\cos(x-y) &= \cos(x)\cos(y) + \sin(x)\sin(y)
\end{aligned}$$

   (a) Use the first two identities to get $\sin(x)\cos(y)$ in terms of $\sin(x+y)$ and $\sin(x-y)$.

   (b) Use the next two identities to get $\cos(x)\cos(y)$ in terms of $\cos(x+y)$ and $\cos(x-y)$.

   (c) Do the same for $\sin(x)\sin(y)$.

2. Find $\displaystyle\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx$, given that m and n are positive integers.

3. We've done a few integrals with sines and cosines. Fill in this table:

	1	$\sin(nx)$	$\cos(nx)$
1			
$\sin(mx)$			
$\cos(mx)$			

   with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

4. (This problem appeared on a Winter, 2010 Math 116 Exam) Calculate the following improper integrals.

   (a) $\displaystyle\int_0^2 \frac{3}{x^{1/3}}\,dx$ (b) $\displaystyle\int_0^2 \frac{e^{-1/x}}{x^2}\,dx$

5. Recently we found that the probability that a novice player's score in the game of Continuous Darts is less than some value x is

$$P(x) = 2x - x^2.$$

   That's called the **cumulative distribution function** or **CDF** of the score.

   (a) Use $P(x)$ to find the probability that the score is between 1/3 and 2/3.

   (b) How would you use $P(x)$ to find the probability that a score is between a and b?

   (c) Hmmm, that answer reminds you of the First Fundamental Theorem of Calculus, I bet. Can you write it as an integral?

   The derivative of $P(x)$ is called the **probability density function** or **PDF** of the score. Let's call if $p(x)$.

6. Find the mean score of Continuous Darts by computing the integral

$$\int_{-\infty}^{\infty} xp(x)\,dx.$$

7. (This problem appeared on a Winter, 2003 Math 116 exam) Fred likes to juggle. So does Jason. The number of minutes Fred can juggle five balls without dropping one is a random variable, with probability density function $f(t) = 0.8e^{-0.8t}$. Similarly, the function $j(t) = 1.5e^{-1.5t}$ describes Jason's skill. Here t is time *in minutes*.

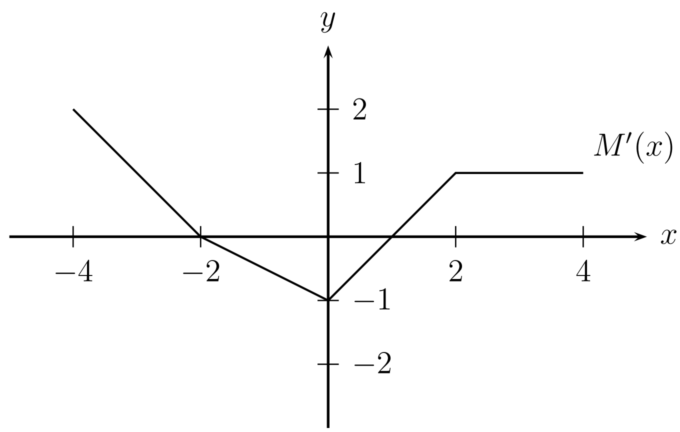
   (a) Find $\int_0^\infty f(t)\,dt$.

   (b) What percentage of Jason's juggling attempts are "embarrassing," meaning they last for 10 seconds or less?

   (c) How long can Fred juggle, on average?

   (d) Who is the better juggler? Give a good reason for your decision.

8. Here is the graph of the *derivative* of the continuous function $M(x)$. Using the fact that $M(-4) = -2$, sketch the graph of $M(x)$. Give the coordinates of all critical points, inflection points, and endpoints.



9. (Winter, 2003) Below are the graphs of several functions $f(x)$, $g(x)$, $h(x)$, $i(x)$, $j(x)$, and $k(x)$. Do not assume that the y-axis scales on these graphs are equal or even comparable. We have calculated LEFT(6), RIGHT(6), TRAP(6), and MID(6) for four of these six functions. Label each column with the name of the function estimated in that column.

Function:				
LEFT(6):	64.2	.328	.255	80.0
RIGHT(6):	65.8	.444	.421	80.0
TRAP(6):	65.0	.386	.338	80.0
MID(6):	65.0	.388	.331	80.0