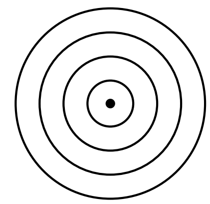
Douglass Houghton Workshop, Section 2, Tue 01/28/20

Worksheet Et tu, Brute?

1. Find $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$, given that m and n are positive integers.

2. Consider a game of "continuous darts." The board is circular, as you expect, with radius 1. The goal is to get as close to the middle as possible. If a dart lands a distance r from the bullseye, its score is $1 - r$. (So every number between 0 and 1 is a possible score.)



A novice player throws a dart which lands randomly somewhere on the board. That means that for any region R on the board,

$$\text{Prob(dart lands in } R) = \frac{\text{area of } R}{\text{area of board}}.$$

(a) Fill in the table with the probabilities that the dart scores **below** the given value.

x	0	1/4	1/2	3/4	1
Prob(score $< x$)					

(b) Let x be any number. Find the probability that the score is less than x.

(c) Find the median score.

3. The function you found in (2b) above is called the **cumulative distribution function** or **CDF** of the score. Let's call it $P(x)$.

(a) Use $P(x)$ to find the probability that the score is between 1/3 and 2/3.

(b) How would you use $P(x)$ to find the probability that a score is between a and b?

(c) Hmmm, that answer reminds you of the First Fundamental Theorem of Calculus, I bet. Can you write it as an integral?

The derivative of $P(x)$ is called the **probability density function** or **PDF** of the score. Let's call if $p(x)$.

4. A while back we showed that if a parabola $y = p(x)$ goes through the three points $(-1, R)$, $(0, S)$, and $(1, T)$, then $\int_{-1}^{1} p(x)\,dx = \frac{1}{3}(R + 4S + T)$. Suppose now that a parabola $y = q(x)$ goes through the points (x_0, y_0), (x_1, y_1), and (x_2, y_2), with $x_0 = x_1 - h$ and $x_2 = x_1 + h$. (So x_1 is midway between the other x's.)

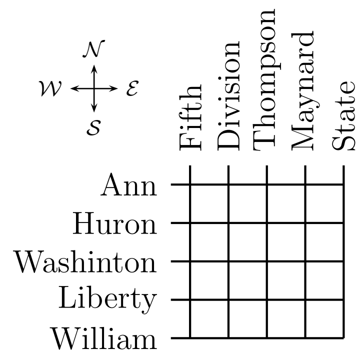
(a) Find a linear substitution that makes $\int_{x_0}^{x_2} q(x)\,dx$ into an integral from -1 to 1. Is the integrand still a parabola?

(b) Use our previous result to calculate the area under $q(x)$ from x_0 to x_2.

5. (This problem appeared on a Winter, 2010 Math 116 Exam) Calculate the following improper integrals.

(a) $\displaystyle\int_0^2 \frac{3}{x^{1/3}}\,dx$ (b) $\displaystyle\int_0^2 \frac{e^{-1/x}}{x^2}\,dx$

6. A small section of downtown Ann Arbor is shown to the right. Copy the map onto the board.



(a) Suppose Myrka lives at the corner of Washington and Thompson, and she needs to get to class at Mason Hall, which is at State and William. She doesn't want to walk out of her way, so she will only go east and south. Still, she has some choices. How many ways are there to get to class?

(b) Interesting, I wonder what that number means? Write your answer to part (a) at the corner of Washington and Thompson. Now pick a different starting corner, and figure out how many ways there are to get to class from there. Repeat, writing your answers on the board at the relevant corner.

(c) What's the pattern?

(d) Explain why the pattern must continue to hold, no matter how big the city is.

7. (Winter, 2003) Below are the graphs of several functions $f(x)$, $g(x)$, $h(x)$, $i(x)$, $j(x)$, and $k(x)$. Do not assume that the y-axis scales on these graphs are equal or even comparable. We have calculated LEFT(6), RIGHT(6), TRAP(6), and MID(6) for four of these six functions. Label each column with the name of the function estimated in that column.

Function:				
LEFT(6):	64.2	.328	.255	80.0
RIGHT(6):	65.8	.444	.421	80.0
TRAP(6):	65.0	.386	.338	80.0
MID(6):	65.0	.388	.331	80.0