Douglass Houghton Workshop, Section 2, Tue 01/21/20

Worksheet Catnip

1. Let's practice some substitution.

   (a) $\int z(z+3)^{1/3}\,dz$

   (b) $\int \dfrac{dx}{2+2\sqrt{x}}$

   (c) $\int_{-3}^{0} (z+2)\sqrt{1-z}\,dz$

   (d) $\int_{4}^{12} \dfrac{3x-2}{\sqrt{2x+1}}\,dx$

2. Let's practice some integration by parts.

   (a) $\int x^2 e^x\,dx$

   (b) $\int \ln x\,dx$

   (c) $\int e^x \sin x\,dx$

   (d) $\int_0^1 \tan^{-1}(x)\,dx$ Hint: $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$

3. Suppose we want to compute $\displaystyle\int \frac{2x+5}{x^2-2x-3}\,dx$.

   (a) Factor the denominator into something like $(x-\alpha)(x-\beta)$.

   (b) Now reverse the process of finding a common denominator. That is, imagine the integrand can be written as
   $$\frac{A}{x-\alpha} + \frac{B}{x-\beta}$$
   for some constants A and B. Find what A and B have to be to make that the same as $\frac{2x+5}{x^2-2x-3}$.

   (c) Finally, rewrite the integral using the sum you found, and use substitution to solve it.

4. Use one of the trig identities in the front of your textbook to compute $\int \sin^2(x)\,dx$.

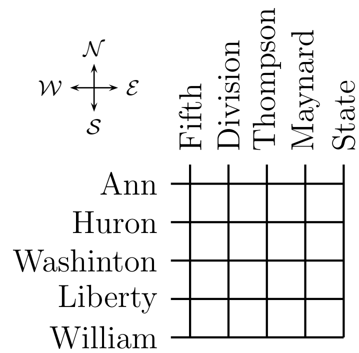
5. The **expectation** of a particular bet on a particular game is the average amount you'll win if you play many times.

   (a) Suppose among a certain group of people, 54% get 1 scoop of ice cream, 32% get 2 scoops, and 14% get 3 scoops. What is the average number of scoops per person?

   (b) If you bet \$1 on red in Roulette, there are 2 possbile outcomes. Write down the probabilities and payoffs for each, and find the exected payoff.

   (c) Find the expectation of The Michigan Lottery's non-boxed pick-3 game. The cost of a ticket is \$1, and if your number comes up you can turn in the ticket for \$500.

6. A small section of downtown Ann Arbor is shown to the right. Copy the map onto the board.



   (a) Suppose Myrka lives at the corner of Washington and Thompson, and she needs to get to class at Mason Hall, which is at State and William. She doesn't want to walk out of her way, so she will only go east and south. Still, she has some choices. How many ways are there to get to class?

   (b) Interesting, I wonder what that number means? Write your answer to part (a) at the corner of Washington and Thompson. Now pick a different starting corner, and figure out how many ways there are to get to class from there. Repeat, writing your answers on the board at the relevant corner.

   (c) What's the pattern?

   (d) Explain why the pattern must continue to hold, no matter how big the city is.

7. Currently 95% of Michigan kindergarteners have been vaccinated for measles. The measles vaccine is 93% effective, meaning that 7% of vaccinated children who are exposed to the disease will contract it, and the rest will not. That contrasts with a 10% immunity among unvaccinated children.

   (a) Suppose that all children in the community are exposed to the measles vaccine, and fill in the following table of possibilities. For instance, the upper-left corner is the probability that a randomly-chosen child is vaccinated *and* contracts measles.

		Vaccinated?	
		Yes	No
Gets measles?	Yes		
	No		

   (b) What proportion of the students who contract measles were vaccinated?

   (c) What does that mean about whether you should vaccinate your child?

8. A while back we showed that if a parabola $y = p(x)$ goes through the three points $(-1, R)$, $(0, S)$, and $(1, T)$, then $\int_{-1}^{1} p(x)\,dx = \frac{1}{3}(R + 4S + T)$. Suppose now that a parabola $y = q(x)$ goes through the points (x_0, y_0), (x_1, y_1), and (x_2, y_2), with $x_0 = x_1 - h$ and $x_2 = x_1 + h$. (So x_1 is midway between the other x's.)

   (a) Find a linear substitution that makes $\int_{x_0}^{x_2} q(x)\,dx$ into an integral from -1 to 1. Is the integrand still a parabola?

   (b) Use our previous result to calculate the area under $q(x)$ from x_0 to x_2.