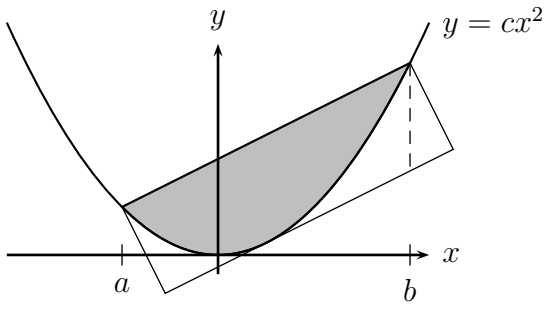
Douglass Houghton Workshop, Section 2, Tue 01/14/20

Worksheet Batrachomyomachia

1. Last time we found that:

$$\text{SHADED AREA} = c(b-a)^3/6$$
$$\text{SLOPE OF LINES} = c(a+b)$$
$$\text{EQUATION OF TOP LINE}: y = c(a+b)x - abc$$



For reasons unexplained, we want the area of the box containing the shaded area.

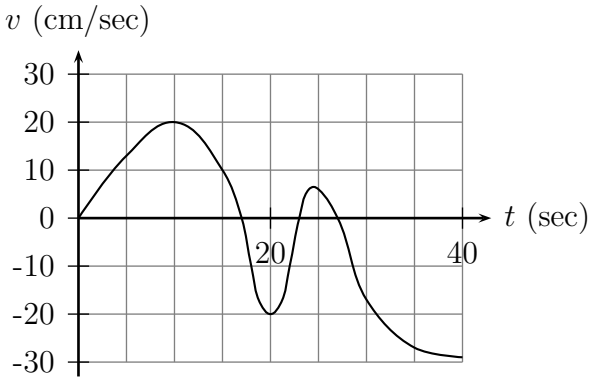
   (a) The bottom line is tangent to the curve. Find the point of tangency and the equation of the tangent line.

   (b) Imagine slicing a triangle off the right side of the box, along the dashed line, and gluing it onto the left side of the box. What shape do you have? Does this suggest a way to find the area of the box?

   (c) Find the area of the box.

2. (This is derived from Problem 47 on page 295 of your book.)

A mouse is trapped in a psychologist's experiment. She moves back and forth in a straight tunnel. The cruel experimenter attracts the mouse with bits of cheese at one end or the other. Sometimes he also puts a frog in the tunnel to scare the mouse away, because mice are terrified of frogs. The graph of the mouse's velocity, v, is given to the right, with a positive velocity corresponding to motion toward the right end.



Tell the story from the mouse's point of view. You might write it as a timeline, explaining what happened when. Make up explanations for all the significant features of the graph.

3. The breathing of a frog is cyclic, and when it is relaxed (because no mice are around), the time from beginning of inhalation to end of exhalation is about 5 seconds. The maximum rate of air flow into the lungs is about 50 milliliters per second.

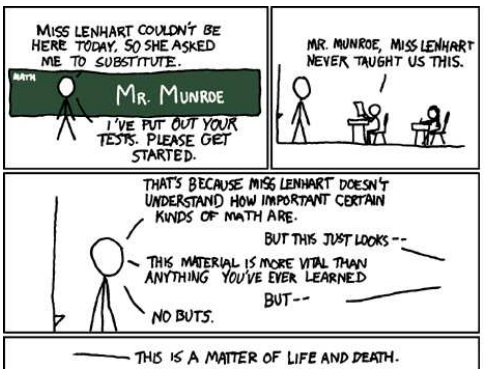
   (a) Write a trigonometric function that models the rate of air flow into the lungs.

   (b) Use this function to find the maximum amount of inhaled air in the lungs.

4. Find $\dfrac{d}{dx}\displaystyle\int_{\cos x}^{3} e^{t^2}\,dt$.

5. Find $\dfrac{d}{dx}\displaystyle\int_{g(x)}^{h(x)} f(t)\,dt$.

6.



The velociraptor spots you 40 meters away and attacks, accelerating at 4 m/s^2 up to its top speed of 25 m/s. When it spots you, you begin to flee, quickly reaching your top speed of 6 m/s. How far can you get before you're caught and devoured?

7. Find the interval on which the graph of $f(x) = \int_0^x \frac{1}{1+t+t^2}\,dt$ is concave up.

8. The Michigan Lottery offers several exciting and fun ways to spend money. Let's calculate the odds of one of them.

   Daily 3 Three bins, numbered 1, 2, and 3, each contain ten ping-pong balls, numbered 0 through 9. A ball is chosen from each bin, so that the result of the drawing is a 3-digit number. Players likewise choose a 3-digit number to play.

   (a) What is the probability of getting all three digits correct?

   (b) You can also play your numbers "boxed". That means that if you match the three digits *in any order*, you win. What is the probability of winning a boxed ticket? Does it depend on what numbers you play?

9. The **expectation** of a particular bet on a particular game is the average amount you'll win if you play many times.

   (a) Suppose among a certain group of people, 54% get 1 scoop of ice cream, 32% get 2 scoops, and 14% get 3 scoops. What is the average number of scoops per person?

   (b) If you bet \$1 on red in Roulette, there are 2 possbile outcomes. Write down the probabilities and payoffs for each, and find the exected payoff.

   (c) Find the expectation of The Michigan Lottery's non-boxed pick-3 game. The cost of a ticket is \$1, and if your number comes up you can turn in the ticket for \$500.

10. Suppose you have a function $f(x)$. You know:

    - f is a quadratic. That is, $f(x) = ax^2 + bx + c$ for some constants a, b, and c.
    - How to measure $f(-1)$, $f(0)$, and $f(1)$.

    You want to know $\int_{-1}^{1} f(x)\,dx$.

    (a) Let R, S, and T be the values you measure for $f(-1)$, $f(0)$, and $f(1)$. What are R, S, and T in terms of a, b, and c?

    (b) Find a formula for $f(x)$. That is, find a, b, and c in terms of R, S, and T.

    (c) Find $\int_{-1}^{1} f(x)\,dx$ in terms of R, S, and T.