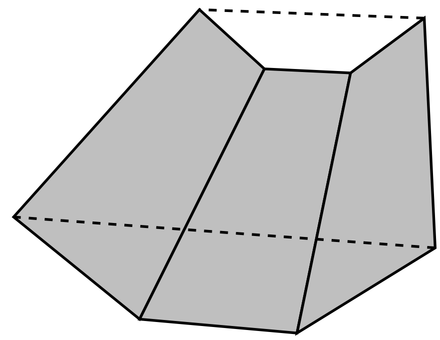
Douglass Houghton Workshop, Section 1, Mon 04/13/20

Worksheet Somewhere, Beyond the Sea



1. The picture to the right shows a section of the Los Angeles river, whose sides are lined with concrete. It is currently full of water, but we need to empty it so we can film a car chase scene for a movie (as in *Terminator 2*, *Grease*, *Gone in 60 Seconds*, *Buckaroo Banzai*, etc.) It is 100 meters long, 17 meters deep, 40 meters wide at the top and 20 meters wide at the bottom. Find the work required to pump all the water up to the top of the river.

   (a) Draw a vertical cross-section of the river (i.e. a trapezoid) on the board. Label the dimensions you know.

   (b) Our slices will be slabs of water parallel to the surface. On your picture, draw a horizontal rectangle representing one end of the slice. Also, make up a variable to define where your slice is, relative to the top or the bottom (you decide). Put your variable on your picture.

   (c) Find the width of the slice in terms of your variable. Hint: it should be a linear function, and you know two points of it.

   (d) Now find these quantities, in the order given. NO INTEGRALS UNTIL YOU FINISH THIS PART!

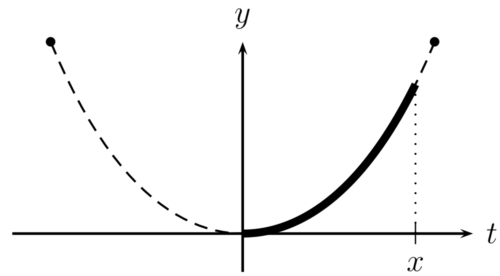
   - length of slice
   - volume of slice
   - mass of slice
   - weight of slice
   - dist to lift slice
   - work to lift slice

   (e) Find the total work it takes to pump out the river. NOW you can do an integral.

2. We've made some progress finding the shape of a hanging chain. If the shape is given by $F(x)$, then by considering forces and arc length we've shown that

$$T_0 F'(x) = \delta g \int_0^x \sqrt{1 + F'(t)^2}\, dt$$

where T_0 is the tension at the bottom of the chain, δ is the mass density of the chain, and g is acceleration due to gravity (all constants). Where to go from here? We'd like to find a formula for $F(x)$.



(a) That thing on the right is begging for you take its derivative. ("Take my derivative!" it cries.) So take the derivative of both sides with respect to x.

(b) Hmmm. No Fs, only F's. And lots of constants. Let $y = F'(x)$, and put all the constants together into one constant. That should make it look better.

(c) What is y when x is 0? Now you have an initial value to go with your differential equation.

(d) Separate the variables and solve the differential equation.