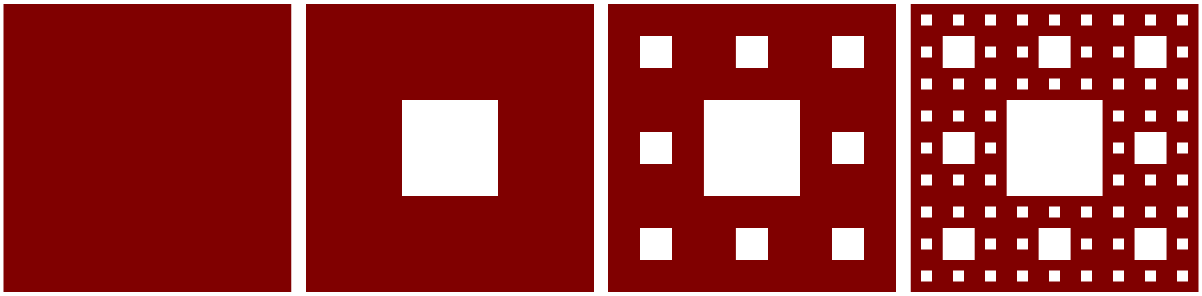
Worksheet Pain and pleasure, like light and darkness, succeed each other

1. (Adapted from a Fall, 2005 Math 116 exam) The Sierpinski Carpet is an example of a mathematical object called a fractal. To construct it, start with a 1×1 red square (stage 0). Then,

   - In stage 1, remove the center $\left(\frac{1}{3} \times \frac{1}{3}\right)$ square,
   - In stage 2, remove the centers of the remaining 8 squares,
   - In stage 3, remove the centers of all the remaining squares,

   and so on, for infinitely many stages. The figure below shows stages 0 through 3.



   (a) Fill in the table below with data about the first few stages of the process:

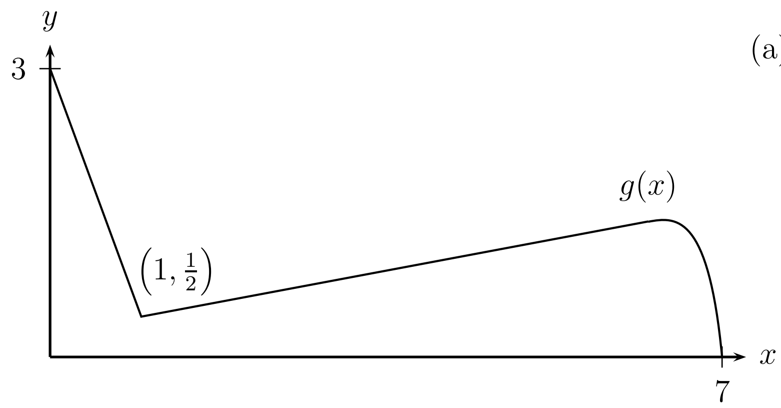
Stage	# squares removed	size of each removed square
1		
2		
3		

   (b) What would the nth entry in the table say? How much area is removed in the nth stage of construction?

   (c) How much area has been removed from stage 0 to stage n?

   (d) How much area is left after an infinite number of stages?

2. (From a Fall, 2015 Math 116 Exam) The graph of part of a function $g(x)$ is below.



(a) A thumbtack has the shape of the solid obtained by rotating the region bounded by $y = g(x)$, the x-axis, and the y-axis, about the y-axis. Find an expression involving integrals that gives the volume of the thumbtack.

(b) A doorknob has the shape of the solid obtained by rotating the same region about the x-axis. Find an expression involving integrals that gives the volume of the door knob.

3. (Adapted from a Winter, 2010 exam problem)

(a) Find the first four nonzero terms of the Taylor series for $\ln(1+x)$ about $x = 0$.

(b) Find the first three nonzero terms of the Taylor series for $g(x) = \ln\left(\dfrac{1+x}{1-x}\right)$ about $x = 0$. Hint: Rules of logarithms.

(c) Find the exact value of the sum of the series $2\left(\frac{3}{4}\right) + \frac{2}{3}\left(\frac{3}{4}\right)^3 + \frac{2}{5}\left(\frac{3}{4}\right)^5 + \cdots$

4. (Fall, 2007) Find the interval of convergence of $\displaystyle\sum_{n=3}^{\infty} \frac{(3-x)^{3n}}{8^n(n-2)}$.