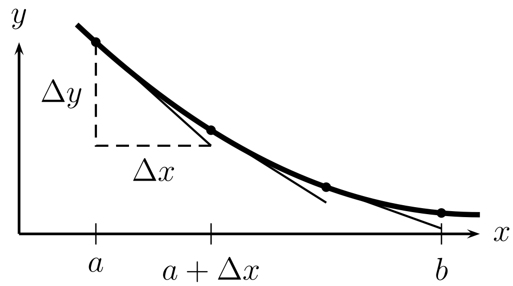
**Douglass Houghton Workshop, Section 1, Wed 03/11/20**

# Worksheet Kimono

1. How can we compute the length of a curve $y = f(x)$? Consider cutting it up into small pieces, and approximating each piece with a line segment, as in the picture below.



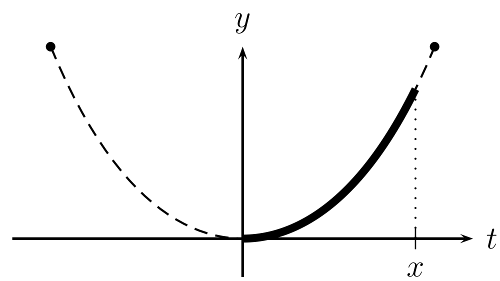

   (a) How long is the first piece? It is tangent to the curve at $a$.

   (b) How long is the $i$th piece?

   (c) Write the left-hand Riemann sum for the length of the curve from $a$ to $b$.

   (d) Now make it into an integral, which will be our formula for arc length.

2. In our quest to determine the shape of a hanging chain, we have found that the forces on a portion of the chain obey a certain relationship: if $m(x)$ is the mass of the chain between the middle and position $x$, $T_0$ is the tension in the chain at the bottom, and $y = F(x)$ is the shape of the chain, then in order to make the forces balance we must have:
$$\frac{m(x)g}{T_0} = F'(x).$$




   (a) How could you calculate $m(x)$ if you knew $F(x)$?

   (b) Some of that we know how to do. Use it to modify the equation above. Feel free to combine unknown constants into one, and simplify as much as possible.

3. The symbol $i$ is often used to represent $\sqrt{-1}$. *It is not a real number*, because of course any real number, when squared, is positive, but $i^2 = -1$. Just the same, it is often very useful (not just in math, but in physics and engineering) to form the set of **complex numbers**
$$\{x + iy : x \text{ and } y \text{ are real numbers}\}$$
and then try to do with complex numbers everything we're used to doing with real numbers. (Most things will work, some won't, and some will work better.)

   (a) We know that $i^2 = -1$, so $i^3 = i^2 \cdot i = (-1) \cdot i = -i$. Write down some more powers of $i$ until you have a general formula for $i^n$.

   (b) Use the power series you found in the last problem above to find $\cosh(i\theta)$, where $\theta$ is a real number.

   (c) Find $\sinh(i\theta)$.

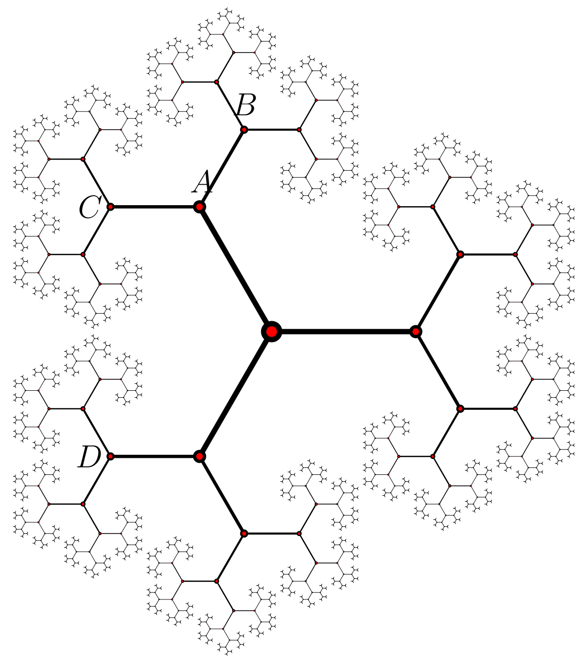
   (d) Add them together to get $e^{i\theta}$. Now you've defined what it means to take a number to an imaginary power!

   (e) Evaluate at $\theta = \pi$.

   (f) Admire your work, with wonder and amazement.

4. Find the full Taylor series for $f(x) = \dfrac{1}{\sqrt{1-4x}}$ about $x = 0$. Also find the radius of convergence.

5. The figure below contains only $120°$ angles. As you move away from the center, the line segments get shorter by a factor $r$. That is, the longest segments (connected to the center) have length 1, the next longest have length $r$, the next longest after that have length $r^2$, etc. Assume the branches keep splitting and splitting, ad infinitum. Most of your answers will be in terms of $r$, but we'll be able to find what $r$ is in part (h). No '...' or '$\sum$' allowed in any of your answers.




(a) Suppose you start at the center and follow the generally northward path. That is, go to $A$, then turn right and go to $B$, then turn left, right, left, etc. How far will you travel after $n$ steps? How far will you travel if you take an infnite number of steps?

(b) If you take the path described in part (a), how far to the north will you have gone when you reach $A$? (That is, how much higher on the page is $A$ than the center?) How far north will you have gone when you reach $B$? When you have gone $n$ steps?

(c) Use the result of part (b) to give the total height of the figure.

(d) Find the distance from the center to the left side of the figure by following the generally northwestward path that goes to $A$, then turns left to $C$, then right, left, right, etc. This time you want the horizontal distance travelled.

(e) Find the distance from the center to the right side of the figure by following a generally northeastward path. Thus find the total width of the figure.

Now of course, the picture could be drawn with any value of $r$. But if $r$ were too large, the figure would overlap itself, and if $r$ were too small, it would hard to see what's going on. The picture above was drawn by using the largest possible value of $r$ which doesn't cause overlap. Thus the path that goes generally southward from $C$ never crosses the path that goes generally northward from $D$, but they do approach the same point.

(f) Find the vertical distance from $C$ to $D$ by using a path through the center.

(g) Find the same distance by considering the southward path from $C$ and the northward path from $D$.

(h) Set them equal and solve for $r$. Do you recognize this number?