Douglass Houghton Workshop, Section 1, Wed 02/19/20

Worksheet Go Placidly Amid the Noise and Haste

With a lot of hard work, we filled the table to the right with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column, and m and n are positive integers.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m=n \\ 0 & \text{otherwise} \end{cases}$

1. Let $h(x) = 5 + 2\cos(x) + \sin(x) - 5\cos(2x) + 3\sin(2x)$.

   (a) Use your calculator to compute:

   $$\int_{-\pi}^{\pi} h(x)\,dx =$$
   $$\int_{-\pi}^{\pi} h(x)\cos(x)\,dx =$$
   $$\int_{-\pi}^{\pi} h(x)\sin(x)\,dx =$$
   $$\int_{-\pi}^{\pi} h(x)\cos(2x)\,dx =$$
   $$\int_{-\pi}^{\pi} h(x)\sin(2x)\,dx =$$

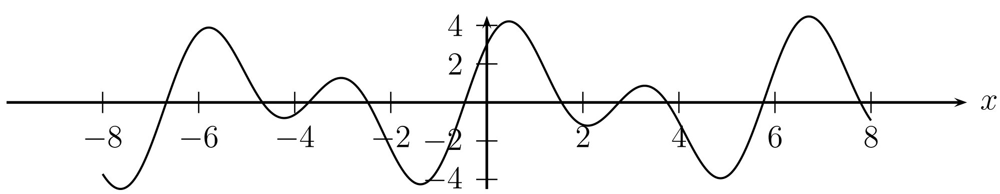
   (b) Explain the results using the table above.

2. Predict what the integrals in (1a) above will be if we change $h(x)$ to

   $$h(x) = 2 + 3\cos(x) - 7\sin(x) - 4\cos(2x) + \sin(2x).$$

3. Generalize: What will those integrals be if

   $$h(x) = a_0 + a_1\cos(x) + b_1\sin(x) + a_2\cos(2x) + b_2\sin(2x).$$

4. 

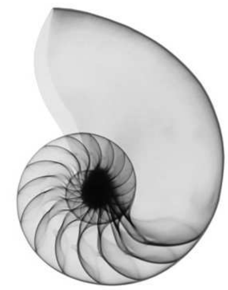
   Suppose $h(x)$ is some function you measure in nature, and its graph looks like the one above. You do some numerical integration and discover that

   $$\int_{-\pi}^{\pi} h(x)\,dx = 0 \qquad \int_{-\pi}^{\pi} h(x)\cos(2x)\,dx = 6.28$$
   $$\int_{-\pi}^{\pi} h(x)\cos(x)\,dx = 3.14 \qquad \int_{-\pi}^{\pi} h(x)\sin(2x)\,dx = 4.71$$
   $$\int_{-\pi}^{\pi} h(x)\sin(x)\,dx = 6.28 \qquad \int_{-\pi}^{\pi} h(x)\cos(nx)\,dx = \int_{-\pi}^{\pi} h(x)\sin(nx)\,dx = 0 \text{ for } n \geq 3.$$

   Can you guess a formula for $h(x)$? Use what you know, and check by graphing your formula on a calculator.

5. Find the probability of winning the pass bet in craps.

6. (6 pts) (This problem appeared on a Winter, 2003 Math 116 exam) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. Show your work on both parts.



   (a) How much volume is enclosed by the last 15 chambers constructed?

   (b) How much volume is enclosed by *all* the chambers? Assume for simplicity that there are infinitely many chambers.

7. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

$$\int_1^\infty \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.

8. (From the Winter, 2011 Math 116 final exam) Determine whether the following series converge or diverge, and justify your answer using one or more convergence tests. No credit without justification.

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{3n-2}{\sqrt{n^5+n^2}}$ (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{n(1+\ln(n))}$

9. Last time it appeared we showed that $e^x = 1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \dfrac{x^4}{4!} + \cdots$. Crazy!

   (a) We got that series by considering derivatives and plugging in $x = 0$. See if you can deduce a series for $\cos(x)$ the same way, by starting with

$$\cos(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

   (b) Test it at $x = \pi$, by adding up all the terms through x^{10}. Is it close to what you expect it to be?

   (c) Do the same for $\sin(x)$.

   (d) Now systemetize the result: if we have a function $f(x)$ which has derivatives, how do we find its series? Find formulas for a_0, a_1, etc. in terms of f.