Douglass Houghton Workshop, Section 1, Mon 02/17/20

Worksheet Flamingo

1. Consider the "Puppy Paws" bet in craps. The bet wins on double fives (⚄⚄) and loses on "soft ten" (⚃⚅ or ⚅⚃) and on 7. If something other than a 7 or 10 is rolled, the bet stays through the next roll.

   (a) Draw the addition table below on the board and fill it in.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

   (b) Calculate these probabilities:
   - W = the probability of winning on the first roll.
   - L = the probability of losing on the first roll.
   - C = the probability that the game continues to a second roll.

   (c) Calculate the probability of winning on the *second* roll.

   (d) Calculate the probability of winning on the kth roll.

   (e) Calculate the probability of winning on *one of* the first n rolls.

   (f) Calculate the probability of winning the Puppy Paws bet.

2. Recall that:

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

   (a) Use the first two identities to get $\sin(x)\cos(y)$ in terms of $\sin(x+y)$ and $\sin(x-y)$.

   (b) Use the next two identities to get $\cos(x)\cos(y)$ in terms of $\cos(x+y)$ and $\cos(x-y)$.

   (c) Do the same for $\sin(x)\sin(y)$.

3. Find $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ where m and n are positive integers.

4. Find $\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx$, given that m and n are positive integers.

5. (6 pts) (This problem appeared on a Winter, 2003 Math 116 exam) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. Show your work on both parts.

   (a) How much volume is enclosed by the last 15 chambers constructed?

   (b) How much volume is enclosed by *all* the chambers? Assume for simplicity that there are infinitely many chambers.

6. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

$$\int_1^\infty \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.

7. We learned recently that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

as long as $|x| < 1$. You can think of both sides of the equation as *functions of* x, and so we have the suprising new idea that a common function we're familar with, $\frac{1}{1-x}$, can be expressed as an infinite series, for certain values of x.

Maybe other functions we know and like can also be expressed as series? Suppose that:

$$e^x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

for some constants $a_0, a_1, a_2, \ldots$, for all x.

(a) What must a_0 be? Hint: plug in 0 to both sides.

(b) Take the derivative of both sides. Now deduce a_1.

(c) Repeat to find $a_2, a_3, a_4, \ldots$.

(d) Can it really be true?!? Try to test with the first 10 terms of the series and $x = 1$.