

Worksheet Batrachomyomachia

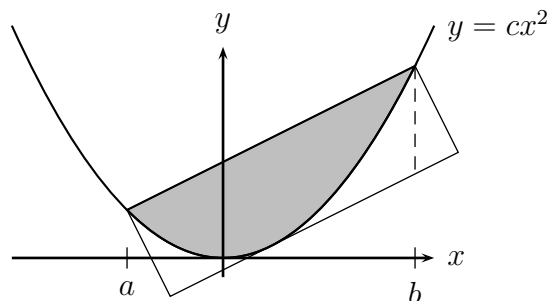
1. Last time we found that:

$$\text{SHADED AREA} = c(b - a)^3/6$$

$$\text{SLOPE OF LINES} = c(a + b)$$

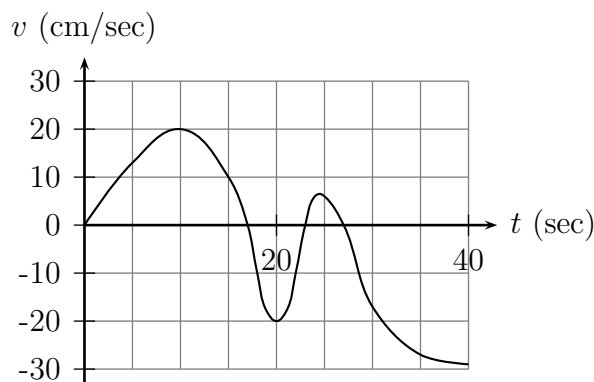
$$\text{EQUATION OF TOP LINE} : y = c(a + b)x - abc$$

For reasons unexplained, we want the area of the box containing the shaded area.



- The bottom line is tangent to the curve. Find the point of tangency and the equation of the tangent line.
 - Imagine slicing a triangle off the right side of the box, along the dashed line, and gluing it onto the left side of the box. What shape do you have? Does this suggest a way to find the area of the box?
 - Find the area of the box.
2. (This is derived from Problem 47 on page 295 of your book.)

A mouse is trapped in a psychologist's experiment. She moves back and forth in a straight tunnel. The cruel experimenter attracts the mouse with bits of cheese at one end or the other. Sometimes he also puts a frog in the tunnel to scare the mouse away, because mice are terrified of frogs. The graph of the mouse's velocity, v , is given to the right, with a positive velocity corresponding to motion toward the right end.

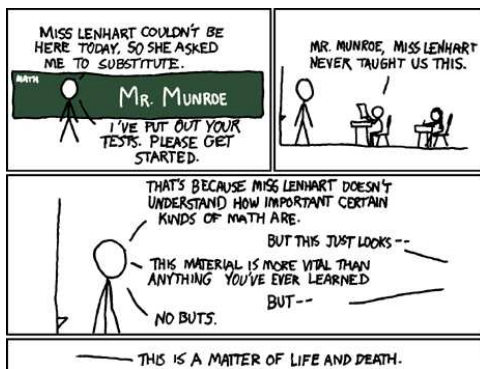


Tell the story from the mouse's point of view. You might write it as a timeline, explaining what happened when. Make up explanations for all the significant features of the graph.

3. The breathing of a frog is cyclic, and when it is relaxed (because no mice are around), the time from beginning of inhalation to end of exhalation is about 5 seconds. The maximum rate of air flow into the lungs is about 50 milliliters per second.
- Write a trigonometric function that models the rate of air flow into the lungs.
 - Use this function to find the maximum amount of inhaled air in the lungs.

4. Find $\frac{d}{dx} \int_{\cos x}^3 e^{t^2} dt$.

5. Find $\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt$.



The velociraptor spots you 40 meters away and attacks, accelerating at 4 m/s^2 up to its top speed of 25 m/s . When it spots you, you begin to flee, quickly reaching your top speed of 6 m/s . How far can you get before you're caught and devoured?



6.

7. Find the interval on which the graph of $f(x) = \int_0^x \frac{1}{1+t+t^2} dt$ is concave up.

8. The Michigan Lottery offers several exciting and fun ways to spend money. Let's calculate the odds of one of them.

Daily 3 Three bins, numbered 1, 2, and 3, each contain ten ping-pong balls, numbered 0 through 9. A ball is chosen from each bin, so that the result of the drawing is a 3-digit number. Players likewise choose a 3-digit number to play.

- (a) What is the probability of getting all three digits correct?
 - (b) You can also play your numbers "boxed". That means that if you match the three digits *in any order*, you win. What is the probability of winning a boxed ticket? Does it depend on what numbers you play?
9. The **expectation** of a particular bet on a particular game is the average amount you'll win if you play many times.
- (a) Suppose among a certain group of people, 54% get 1 scoop of ice cream, 32% get 2 scoops, and 14% get 3 scoops. What is the average number of scoops per person?
 - (b) If you bet \$1 on red in Roulette, there are 2 possible outcomes. Write down the probabilities and payoffs for each, and find the expected payoff.
 - (c) Find the expectation of The Michigan Lottery's non-boxed pick-3 game. The cost of a ticket is \$1, and if your number comes up you can turn in the ticket for \$500.
10. Suppose you have a function $f(x)$. You know:
- f is a quadratic. That is, $f(x) = ax^2 + bx + c$ for some constants a , b , and c .
 - How to measure $f(-1)$, $f(0)$, and $f(1)$.

You want to know $\int_{-1}^1 f(x) dx$.

- (a) Let R , S , and T be the values you measure for $f(-1)$, $f(0)$, and $f(1)$. What are R , S , and T in terms of a , b , and c ?
- (b) Find a formula for $f(x)$. That is, find a , b , and c in terms of R , S , and T .
- (c) Find $\int_{-1}^1 f(x) dx$ in terms of R , S , and T .