Douglass Houghton Workshop, Section 2, Tue 04/16/19

Worksheet Rendezvous with Destiny

1. Find the probability of winning the pass bet in craps.

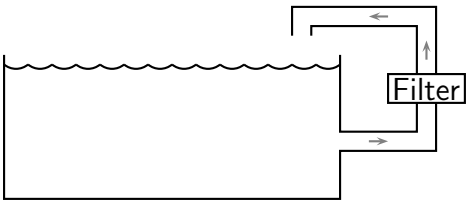
2. (This problem appeared on the Fall, 2007 Math 116 Final Exam. Really!) Newton's law of cooling (or warming) says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Suppose that a thermometer used by a veterinarian to find the temperature of a sick horse obeys Newton's law of cooling. Further suppose that before insertion the thermometer reads 82° F, after one minute it reads 92°, and after another minute it reads 97° F, and that a sudden convulsion unexpectedly destroys the thermometer after the 97° reading. Call the horse's temperature T_h.

   (a) Write a differential equation for the temperature T (a function of time t) of the thermometer. Your equation may involve the constant T_h.

   (b) Solve the differential equation for T to find a general solution for T. Your solution may include undetermined constants such as T_h.

   (c) Use the temperature data to solve for T.

3. (This problem appeared on a Fall, 2010 Math 116 Exam) An aquarium containing 100 liters of fresh water will be filled with a variety of small fish and aquatic plants. A water filter is installed on the tank to help remove the ammonia produced by the decomposing organic matter generated by plants and fish in the aquarium. The filter takes water from the tank at a rate of 20 liters every hour. The water is then filtered and returned to the aquarium at the same rate of 20 liters every hour. Ninety percent of the ammonia contained in the water that goes through the filter is removed. It is estimated that the fish and plants produce 30 mg of ammonia every hour. Assume the ammonia mixes instantly with the water in the aquarium.

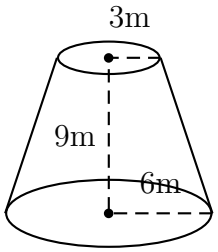


   (a) Let $Q(t)$ be the amount in mg of ammonia in the fish tank t hours after the fish were introduced into the aquarium. Find the differential equation satisfied by $Q(t)$. Include its initial condition.

   (b) Find the amount of ammonia in the fish tank 3 hours after the fish were introduced into the aquarium. Include units.

   (c) What happens to the value of $Q(t)$ in the long run?

4. (From a Winter, 2014 Math 116 Exam) The Nub's Nob Ski Area keeps a massive supply of hot chocolate. The hot chocolate is stored in a container shaped like a cone with the point end removed. The height of the container is 9 meters, and it has lower radius 6 meters and upper radius 3 meters. The hot chocolate has a density of $3000\,\text{kg/m}^3$. Recall the gravitational constant is $g = 9.8\,\text{m/s}^2$.

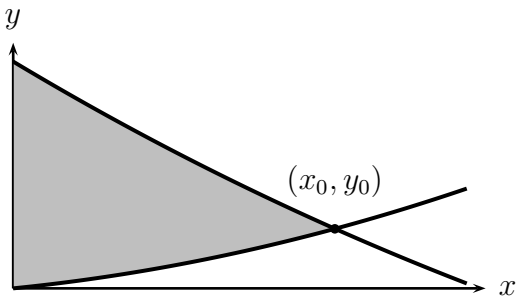


   (a) Write a formula for $r(h)$, the radius of a circular cross section of the container h meters above the base.

   (b) Write a formula in terms of $r(h)$ for the work required to lift a slice of hot chocolate of thickness Δh from height h to the top of the container.

   (c) Write an integral that gives the work required to lift all of the hot chocolate to the top of the container. Then evaluate the integral.

5. (From the Fall, 2010 Math 116 final) The graph shows the area between the graphs of $f(x) = 6\cos(\sqrt{2x})$ and $g(x) = x^2 + x$. Let (x_0, y_0) be the intersection point between the graphs of $f(x)$ and $g(x)$.



   (a) Compute $P(x)$, the function containing the first three nonzero terms of the Taylor series about $x = 0$ of $f(x) = 6\cos(\sqrt{2x})$.

   (b) Use $P(x)$ to approximate the value of x_0.

   (c) Use $P(x)$ and the value of x_0 you computed in the previous question to write an integral that approximates the value of the shaded area. Find the value of this integral.

   (d) Graph $f(x)$ and $g(x)$ in your calculator. Use the graphs to find an approximate value for x_0.

   (e) Write a definite integral in terms of $f(x)$ and $g(x)$ that represents the value of the shaded area. Find its value using your calculator.