Douglass Houghton Workshop, Section 2, Thu 04/11/19

Worksheet Quintessential

1. Previously we found formulas for converting from latitude (ϕ) and longitude (θ) to Cartesian coordinates:

$$\begin{aligned} x &= \rho \cos\phi \cos\theta \\ y &= \rho \cos\phi \sin\theta \\ z &= \rho \sin\phi \end{aligned}$$

Where the origin is the center of the earth and ρ is the distance from the center, which is 1 on the earth's surface.

   (a) How can you find ρ if you know x, y, and z? (Hint: in polar coordinates, how do you find r in terms of x and y?)

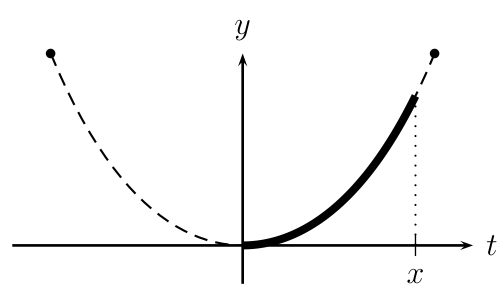
   (b) Likewise find formulas for ϕ and θ in terms of x, y, and z.

2. There is *still* nothing special at latitude 14°38′53″ N, longitude 78°6′28″ W. It's just a point in the ocean. But, if you were to shoot a neutrino from the middle of the Diag (latitude 42°16′36″ N, longitude 83°44′15″ W) to that point, through the earth's crust, its deepest point would be directly under a very interesting place. Find that place, using the Google spreadsheet linked on our homepage.

3. We've made some progress finding the shape of a hanging chain. If the shape is given by $F(x)$, then by considering forces and arc length we've shown that

$$F''(x) = \frac{\delta g}{T_0}\sqrt{1 + F'(x)^2}$$

where T_0 is the tension at the bottom of the chain, δ is the mass density of the chain, and g is acceleration due to gravity (all constants). Where to go from here? We'd like to find a formula for $F(x)$.



   (a) Hmmm. No Fs, only F's. And lots of constants. Let $y = F'(x)$, and put all the constants together into one constant. That should make it look better.

   (b) What is y when x is 0? Now you have an initial value to go with your differential equation.

   (c) Separate the variables and solve the differential equation.

4. (This problem appeared on a Fall, 2004 Math 116 exam.)

   (a) Find the second order Taylor polynomial of $f(x) = \sqrt{4+x}$ for x near 0.

   (b) Find the Taylor series about $x = 0$ of $\sin(2x)$, either from scratch or by using a series you know already.

   (c) Using your answers to parts (a) and (b) and *without computing any derivatives*, find the second order Taylor polynomial that approximates $g(x) = \sqrt{4 + \sin(2x)}$ for x near 0.

   (d) The error in a Taylor polynomial approximation is mostly in the first term omitted, in this case the third order term. So compute that, and give an estimate of the maximum error in the approximation when $-0.1 \leq x \leq 0.1$.

5. (This problem appeared on the Fall, 2007 Math 116 Final Exam. Really!) Newton's law of cooling (or warming) says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Suppose that a thermometer used by a veterinarian to find the temperature of a sick horse obeys Newton's law of cooling. Further suppose that before insertion the thermometer reads 82° F, after one minute it reads 92°, and after another minute it reads 97° F, and that a sudden convulsion unexpectedly destroys the thermometer after the 97° reading. Call the horse's temperature T_h.

   (a) Write a differential equation for the temperature T (a function of time t) of the thermometer. Your equation may involve the constant T_h.

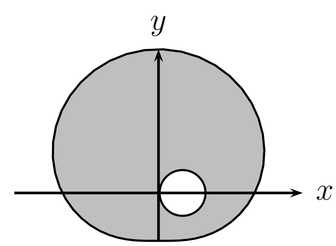
   (b) Solve the differential equation for T to find a general solution for T. Your solution may include undetermined constants such as T_h.

   (c) Use the temperature data to solve for T.

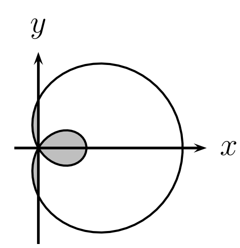
6. (This problem is from a Fall, 2014 Math 116 exam. For some reason, all the exams that term were about robots and chickens.)

   Consider the polar curves

$$r = \cos\theta \qquad \text{and} \qquad r = \sin\theta + 2.$$



   (a) Franklin's robot army occupies the shaded region between these two curves. Find the area occupied by Franklin's robot army.



   (b) Your friend, Kazilla, pours her magic potion on the ground. Suddenly, a flock of wild chickens surrounds you. The chickens occupy the shaded region enclosed within the polar curve $r = 1 + 2\cos\theta$ as shown below. Find the perimeter of the region occupied by the flock of wild chickens.