**Douglass Houghton Workshop, Section 2, Tue 04/09/19**

# Worksheet Poseidon

1. Last time we found that latitudes and longitudes can't be averaged to find a midpoint. This left us bitter and disillusioned, but undaunted. If we could just convert to $(x, y, z)$ coordinates, then we'd be in business.

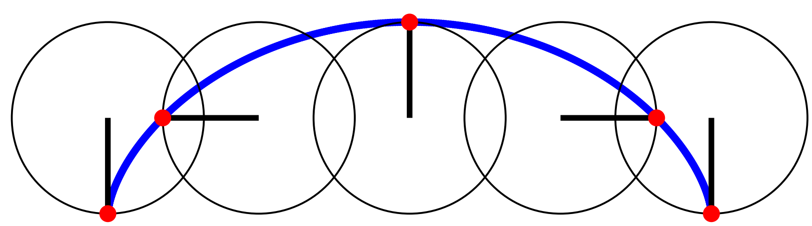
   Write $\phi$ for latitude and $\theta$ for longitude. Define the $(x, y, z)$ coordinate system as:

   - The origin is at the center of the earth.
   - The radius of the earth has length 1.
   - The $x$ axis goes through the point $(\phi = 0, \theta = 90\text{W})$, near the Galapogos Islands.
   - The $y$ axis goes through the point $(\phi = 0, \theta = 0)$, off the coast of Nigeria.
   - The $z$ axis goes through the North Pole.

   (a) Find $z$ in terms of $\phi$ and $\theta$. (One of them doesn't matter.)

   (b) Now find $x$ and $y$. Hint: the plane at latitude $\phi$ intersects the earth in a circle. Draw it on the board. What is its radius?

2. Suppose you are watching Tahmina ride a unicycle in the dark. The only thing you can see is a red reflector which is on the outer edge of the front wheel. As Tahmina moves from left to right, you see the reflector trace out the path below:



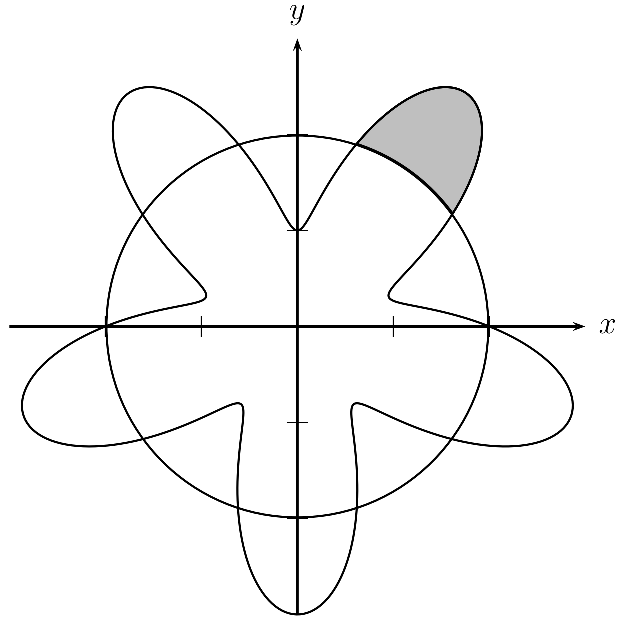
   (a) The wheels in the picture represent snapshots taken every quarter second. The dots are the reflector. Assuming the wheel's center in the first snapshot is $(0, 0)$ and the radius of the wheel is $r$, fill in the table below with the center's position and the reflector's position at time $t$.

| $t$ | 0 sec | 1/4 sec | 1/2 sec | 3/4 sec | 1 sec |
|---|---|---|---|---|---|
| $x_c$ = center's $x$ | 0 | | | | |
| $y_c$ = center's $y$ | 0 | | | | |
| $x_r$ = reflector's $x$ | | | | | |
| $y_r$ = reflector's $y$ | | | | | |

   (b) Find formulas for $x_c$, $y_c$, $x_r$, and $y_r$ in terms of $t$.

   (c) Find the exact distance traveled by the reflector in one minute. No approximations!

3. (Adapted from a Fall, 2010 Math 116 Exam) In the picture to the right, the graphs of $r = 2$ and $r = 2 - \sin(5\theta)$ are shown.




   (a) Write a definite integral that computes the shaded area.

   (b) Compute the area exactly.

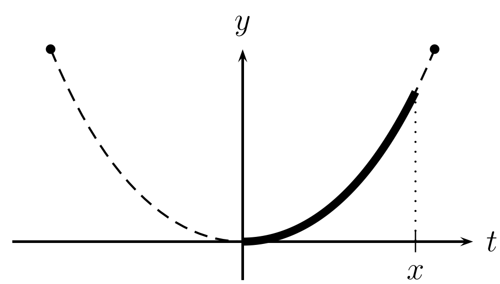
   (c) Write an integral for the length of the boundary of the shaded area.

   (d) Get an approximate answer for that length, using your calculator.

4. We've made some progress finding the shape of a hanging chain. If the shape is given by $F(x)$, then by considering forces and arc length we've shown that

$$T_0 F'(x) = \delta g \int_0^x \sqrt{1 + F'(t)^2}\, dt$$

where $T_0$ is the tension at the bottom of the chain, $\delta$ is the mass density of the chain, and $g$ is acceleration due to gravity (all constants). Where to go from here? We'd like to find a formula for $F(x)$.




   (a) That thing on the right is begging for you take its derivative. ("Take my derivative!" it cries.) So take the derivative of both sides with respect to $x$.

   (b) Hmmm. No $F$s, only $F'$s. And lots of constants. Let $y = F'(x)$, and put all the constants together into one constant. That should make it look better.

   (c) What is $y$ when $x$ is 0? Now you have an initial value to go with your differential equation.

   (d) Separate the variables and solve the differential equation.