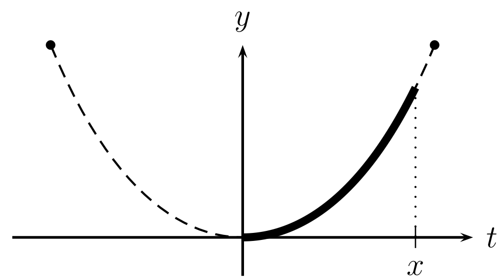
Douglass Houghton Workshop, Section 2, Thu 03/28/19

Worksheet Magnificent

1. Find the full Taylor series for $f(x) = \dfrac{1}{\sqrt{1-4x}}$ about $x = 0$. Also find the radius of convergence.

2. We've made some progress finding the shape of a hanging chain. If the shape is given by $y = F(x)$, then in order for the forces to add up to 0 we must have $F'(x) = m(x)g/T_0$, where

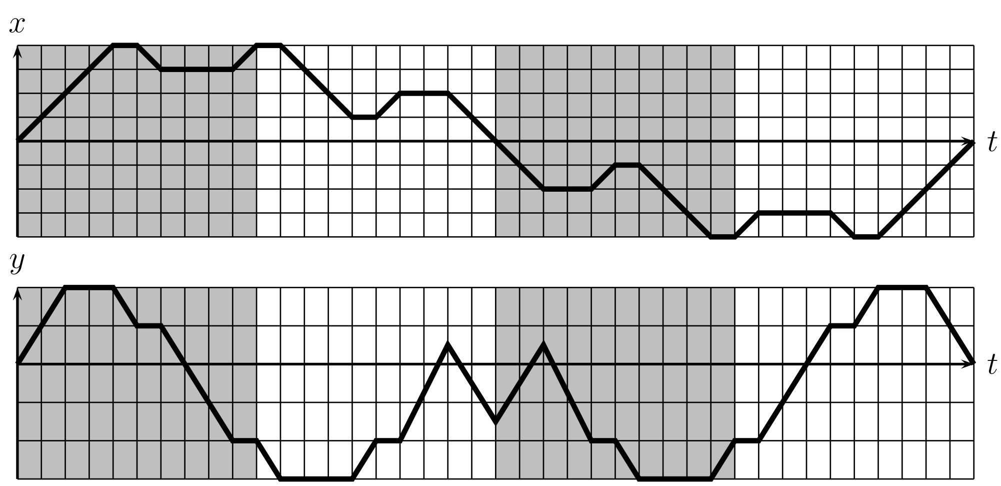


$$\begin{aligned} m(x) &= \text{the mass of shaded part of the chain} \\ g &= \text{acceleraion due to gravity} = 9.8\,\mathrm{m/sec}^2 \\ T_0 &= \text{the tension in the chain at its bottom.} \end{aligned}$$

   (a) Suppose you had a long chain, which was too big to put on a scale. How would you find its mass?

   (b) Now substitute for $m(x)$ in the equation above.

3. Plot the positions of (x, y) given the graphs of $x(t)$ and $y(t)$ below:



4. Write down the Taylor series about $a = 0$ for the following functions, either from memory or by working them out.

   (a) $e^x =$

   (b) $e^{-x} =$

   (c) $\cos(x) =$

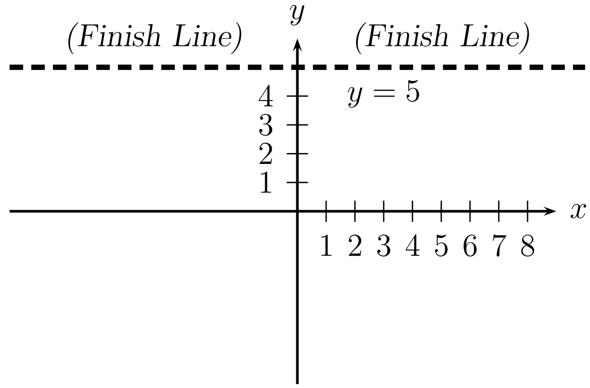
   (d) $\sin(x) =$

   (e) $\cosh(x) = \frac{1}{2}(e^x + e^{-x}) =$

   (f) $\sinh(x) = \frac{1}{2}(e^x - e^{-x}) =$

5. (This problem appeared on a Winter, 2003 Math 116 exam) The newest FOX reality show, "BattleBugs: Clash of the Beetles" begins (at $t = 0$) with eight assorted insects placed randomly on a large mat (the "battlefield", pictured here), on which is marked a "finish line". The producers hoped that the bugs would battle to be first to cross the finish line, but instead they wander around, each according to its nature. The motion of each bug is described by the equations below. Both x and y are measured in inches.

The Battlefield



Hercules Beetle	**Ladybug**	**Tiger Beetle**	**Longhorned Beetle**
$x(t) = \cos(t/2)$ $y(t) = \sin(t/2)$	$x(t) = e^{-t}$ $y(t) = e^{-2t}$	$x(t) = 1 + t$ $y(t) = -1 + 8t$	$x(t) = 3 + t$ $y(t) = 4 - t$
Dung Beetle	**Scarab**	**June Beetle**	**African Ground Beetle**
$x(t) = t$ $y(t) = -2$	$x(t) = 2 - 7t$ $y(t) = -1 - 7t$	$x(t) = 0$ $y(t) = -1$	$x(t) = \sin(t)$ $y(t) = \cos(t)$

Which bug (or bugs)...

(a) move repetitively?

(b) begin closest to the finish line?

(c) move fastest?

(d) will move very slowly (or not at all), in the long run?

(e) will reach the finish line first?

(f) gets the dizziest?

6. The symbol i is often used to represent $\sqrt{-1}$. *It is not a real number*, because of course any real number, when squared, is positive, but $i^2 = -1$. Just the same, it is often very useful (not just in math, but in physics and engineering) to form the set of **complex numbers**

$$\{x + iy : x \text{ and } y \text{ are real numbers}\}$$

and then try to do with complex numbers everything we're used to doing with real numbers. (Most things will work, some won't, and some will work better.)

(a) We know that $i^2 = -1$, so $i^3 = i^2 \cdot i = (-1) \cdot i = -i$. Write down some more powers of i until you have a general formula for i^n.

(b) Use the power series you found in the last problem above to find $\cosh(i\theta)$, where θ is a real number.

(c) Find $\sinh(i\theta)$.

(d) Add them together to get $e^{i\theta}$. Now you've defined what it means to take a number to an imaginary power!

(e) Evaluate at $\theta = \pi$.

(f) Admire your work, with wonder and amazement.