**Douglass Houghton Workshop, Section 2, Tue 03/12/19**

# Worksheet Karma

1. We learned recently that

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

as long as $|x| < 1$. You can think of both sides of the equation as *functions of* $x$, and so we have the suprising new idea that a common function we're familar with, $\frac{1}{1-x}$, can be expressed as an infinite series, for certain values of $x$.

Maybe other functions we know and like can also be expressed as series? Suppose that:

$$e^x = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

for some constants $a_0, a_1, a_2, \ldots$, for all $x$.

   (a) What must $a_0$ be? Hint: plug in 0 to both sides.
   (b) Take the derivative of both sides. Now deduce $a_1$.
   (c) Repeat to find $a_2, a_3, a_4, \ldots$.
   (d) Can it really be true?!? Try to test with the first 10 terms of the series and $x = 1$.

2. Find the probability of winning the pass bet in craps.

3. Between 1980 and 2009, there were 7015 Division I basketball games which were tied at the end of regulation play, resulting in at least one overtime period. Suppose $p$ is the probability that an overtime period ends in a tie.

   (a) How many 2nd overtimes do you expect there were? How many 3rd overtimes? How many $k$th overtimes?
   (b) There were in fact 8568 total overtime periods between 1980 and 2009. Use that to estimate $p$.
   (c) About how many games went 6 or more overtimes, do you guess?

4. Let's prove that a series converges.

   (a) Draw the graph of $y = 1/x^2$ for $x > 0$.
   (b) Draw in some rectangles representing the Right Hand Sum with $\Delta x = 1$ for $\int_1^\infty dx/x^2$.
   (c) Explain why this proves that $\sum_{n=2}^\infty 1/n^2$ converges.
   (d) Of course $\sum_{n=1}^\infty 1/n^2$ is just one bigger than that last sum, so it converes too. But what does it converge to?

5. Use an argument similar to the last one to prove that $\sum_{n=1}^\infty 1/n$ diverges.

6. (From the Winter, 2011 Math 116 final exam) Determine whether the following series converge or diverge, and justify your answer using one or more convergence tests. No credit without justification.

   (a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2^n}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{3n-2}{\sqrt{n^5+n^2}}$ (c) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{1}{n(1+\ln(n))}$

7. (Fall, 2014) Prove whether these series converge or diverge:

   (a) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n\ln(n)}{n}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{n}{\sqrt{n^3+2}}$

8. Prove whether the following improper integrals converge or diverge.

   (a) $\displaystyle\int_3^{\infty}\frac{\ln(x)}{x^2}\,dx$ (b) $\displaystyle\int_0^{\infty}\frac{3}{4x^2+5\sqrt{x}}\,dx$