Douglass Houghton Workshop, Section 2, Thu 02/28/19

Worksheet Just Do It

With a lot of hard work, we filled the table to the right with the values of $\int_{-\pi}^{\pi} f\,dx(x)g(x)\,dx$, where f is the row and g is the column, and m and n are positive integers.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$

The implication was that for a function of the form

$$\begin{aligned} f(x) = a_0 + a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x)\ +\ \cdots \\ + b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x)\ +\ \cdots \end{aligned} \tag{1}$$

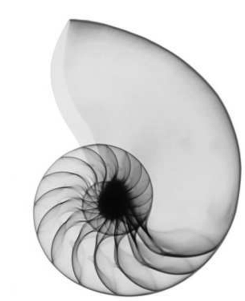
integrating against a sine or cosine function makes almost all the terms 0, so for $n \geq 1$:

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx, \quad \text{and} \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx.$$

Writing a function in the form of Equation (1) is called finding the **Fourier Series** of the function.

1. For reasons unexplained, let's approximate the function $f(x) = x^2$ by its Fourier series.
   (a) The b_n are all 0. Why?
   (b) Find a_0.
   (c) Find a_n for $n \geq 1$. Hint: Ladder Method.

2. (6 pts) (This problem appeared on a Winter, 2003 Math 116 exam) The chambered nautilus builds a spiral sequence of closed chambers. It constructs them from the inside out, with each chamber approximately 20% larger (by volume) than the last. (The large open section at the top is not a "chamber.") The largest chamber is 9 cubic inches. Show your work on both parts.



   (a) How much volume is enclosed by the last 15 chambers constructed?
   (b) How much volume is enclosed by *all* the chambers? Assume for simplicity that there are infinitely many chambers.

3. Find the probability of winning the pass bet in craps.

4. (Adapted from a Fall, 2003 Math 116 Exam)

(a) Express the number

$$.135135135\overline{135}$$

as the sum of a geometric series.

(b) Use the infinite geometric series formula to express that same number as a fraction in lowest terms.