Douglass Houghton Workshop, Section 2, Tue 02/19/19

Worksheet Giraffe

1. Last time we found some identites for converting products of trig functions into sums:

$$\sin(x)\cos(y) = (\sin(x+y) + \sin(x-y))/2$$
$$\cos(x)\cos(y) = (\cos(x-y) + \cos(x+y))/2$$
$$\sin(x)\sin(y) = (\cos(x-y) - \cos(x+y))/2$$

Use those to finish filling in this table:

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0		
$\cos(mx)$	0		

with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

2. A spaceship seeks to reach a height H above the surface of the earth. The force of gravity at any time is

$$\boxed{F_g = G\frac{Mm}{r^2}}$$

$G =$ The universal gravitational constant
$M =$ The mass of the earth
$m =$ The mass of the spaceship
$r =$ The distance from the spaceship to the center of the earth.

(a) How much work will it take to raise the spaceship from the surface of the earth to a point H meters above the surface? Use R for the radius of the earth. Don't assume gravity is constant as the ship moves upward!

(b) How much work would it take to push the spaceship entriely beyond the reach of Earth's gravity? (Let $H \to \infty$.)

(c) If the ship is travelling at velocity v, it will have kinetic energy $\frac{1}{2}mv^2$. That energy will be converted into work to move the ship upward. What speed must the ship be going near the surface to leave the earth's gravity well? This is the earth's *escape velocity*.

(d) Look up the values of G, M, and R, and get a numerical answer in miles per second.

3. Consider the "Hard Eight" bet in craps. The bet wins on double fours (⚃⚃) and loses on "soft eight" (⚁⚅ or ⚂⚄) and on 7. If something other than a 7 or 8 is rolled, the bet stays through the next roll.

   (a) Draw the addition table below on the board and fill it in.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

   (b) Calculate these probabilities:
   - W = the probability of winning on the first roll.
   - L = the probability of losing on the first roll.
   - C = the probability that the game continues to a second roll.

   (c) Calculate the probability of winning on the *second* roll.

   (d) Calculate the probability of winning on the kth roll.

   (e) Calculate the probability of winning on *one of* the first n rolls.

   (f) Calculate the probability of winning the hard-eight bet.

4. We have a new tool for evaluating limits, called *L'Hôpital's Rule*. It allows us to resolve some limits we couldn't do last fall. In particular, we can prove our favorite result, about Michael Phelps's towel.

   (a) Recall our model of toweling: water spreads out evenly over Michael and the towel. So if Michael has area $1\,\mathrm{m}^2$ and starts with wetness W, after using a towel of size T his new wetness will be

$$W\left(\frac{1}{1+T}\right).$$

   Suppose instead he divides his towel of size T into n parts. How wet will he be if he starts with 1 liter of water on him?

   (b) What happens if he divides the towel into more and more pieces? Let

$$L = \lim_{n\to\infty}(\text{the formula you found in part (4a)}).$$

   Take the ln of both sides of the equation above. It's OK to move the ln inside the limit, because ln is a continuous function.

   (c) Let $p = 1/n$. As $n \to \infty$, $p \to 0$. So rewrite your limit with p's instead of n's.

   (d) In order to use L'Hôpital's Rule, you need to have something of the form $0/0$ or ∞/∞. So get you limit in that form, and resolve it.