Douglass Houghton Workshop, Section 2, Thu 02/14/19

Worksheet Fate, or Folly?

1. Former DHSP students Marlee and Terren have a complicated relationship. Each influences how attracted the other is. Let

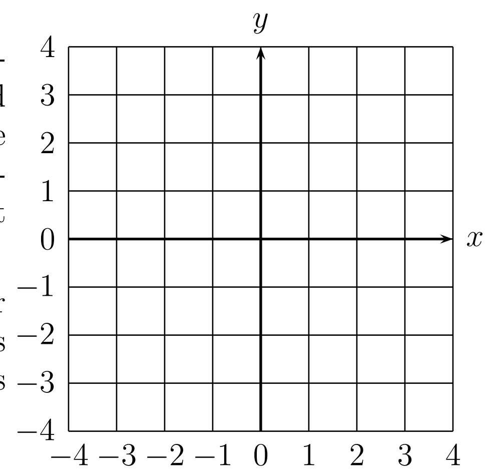
$$x = \text{How attracted Marlee is to Terren}$$
$$y = \text{How attracted Terren is to Marlee}$$

and suppose that the two are related by the differential equations

$$\frac{dx}{dt} = 2 - x - y \qquad \text{and} \qquad \frac{dy}{dt} = x - 1.$$

(a) Use the differential equations to describe the characters in this story. Questions to ask: What kind of a guy is Terren? What strategy can Marlee use to attract him? What happens to Marlee when Terren begins to like her? How should he act to attract her?

(b) Now draw a grid on the board, and at each corner draw a little arrow in the direction that (x, y) is moving. (You'll probably want to find dy/dx.) This is called a *slope field* for the system.



(c) What happens in the long run?

(d) Now write the story of Marlee and Terren's relationship, being true to their characters and the differential equations.

2. Recall that:

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$$
$$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$$
$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$
$$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$$

(a) Use the first two identities to get $\sin(x)\cos(y)$ in terms of $\sin(x+y)$ and $\sin(x-y)$.

(b) Use the next two identities to get $\cos(x)\cos(y)$ in terms of $\cos(x+y)$ and $\cos(x-y)$.

(c) Do the same for $\sin(x)\sin(y)$.

3. We've done a few integrals with sines and cosines. Fill in this table:

	1	$\sin(nx)$	$\cos(nx)$
1			
$\sin(mx)$			
$\cos(mx)$			

with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

4. Consider the "Hard Eight" bet in craps. The bet wins on double fours (⚃⚃) and loses on "soft eight" (⚁⚅ or ⚂⚄) and on 7. If something other than a 7 or 8 is rolled, the bet stays through the next roll.

(a) Draw the addition table below on the board and fill it in.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

(b) Calculate these probabilities:

- W = the probability of winning on the first roll.
- L = the probability of losing on the first roll.
- C = the probability that the game continues to a second roll.

(c) Calculate the probability of winning on the *second* roll.

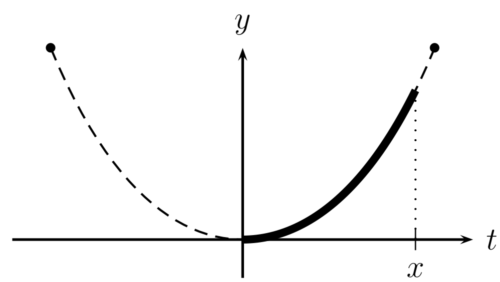
(d) Calculate the probability of winning on the kth roll.

(e) Calculate the probability of winning on *one of* the first n rolls.

(f) Calculate the probability of winning the hard-eight bet.

5. In our quest to determine the shape of a hanging chain, we have found that the forces on a portion of the chain obey a certain relationship: if $m(x)$ is the mass of the chain between the middle and position x, T_0 is the tension in the chain at the bottom, and $y = F(x)$ is the shape of the chain, then in order to make the forces balance we must have:

$$\frac{m(x)g}{T_0} = F'(x).$$



(a) How could you calculate $m(x)$ if you knew $F(x)$?

(b) Some of that we know how to do. Use it to modify the equation above. Feel free to combine unknown constants into one, and simplify as much as possible.

6. The picture to the right shows a section of the Los Angeles river, whose sides are lined with concrete. It is currently full of water, but we need to empty it so we can film a car chase scene for a movie (as in *Terminator 2*, *Grease*, *Gone in 60 Seconds*, *Buckaroo Banzai*, etc.) It is 100 meters long, 17 meters deep, 40 meters wide at the top and 20 meters wide at the bottom. Find the work required to pump all the water up to the top of the river.