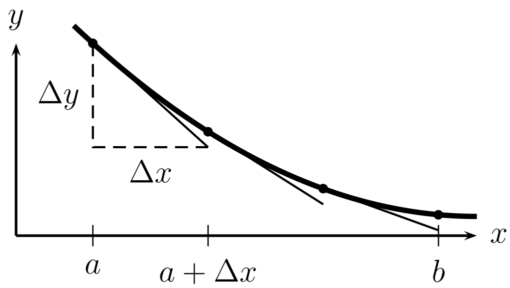
Douglass Houghton Workshop, Section 2, Tue 01/29/19

Worksheet Et tu, Brute?

1. How can we compute the length of a curve $y = f(x)$? Consider cutting it up into small pieces, and approximating each piece with a line segment, as in the picture below.



   (a) How long is the first piece? It is tangent to the curve at a.

   (b) How long is the ith piece?

   (c) Write the left-hand Riemann sum for the length of the curve from a to b.

   (d) Now make it into an integral, which will be our formula for arc length.

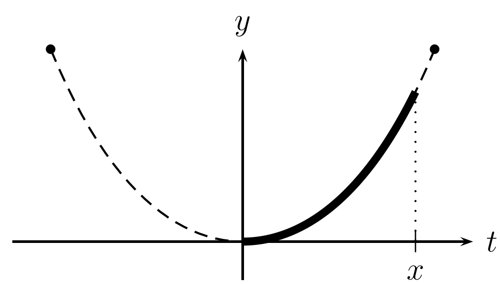
2. Compute the length of the ladder curve, $x^{2/3} + y^{2/3} = 1$.

3. Last time we enumerated the forces on the piece of hanging chain shown here: gravity (mg), leftward tension (T_0) and tension pulling up and to the right (T).

   (a) T can be split into horizontal and vertical components. What are their sizes? (Remember the forces must sum to 0.)

   (b) Let $y = F(t)$ be the shape of the chain. What does the fact that T is pulling in the direction of the chain tell you about the slope of F?

   (c) T_0 is constant as x changes. But the force of gravity is not. Why? How could we find the weight of the piece of chain if we knew $F(t)$?

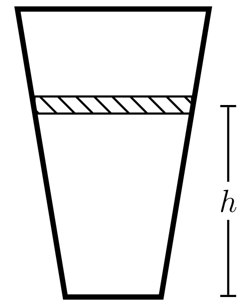


4. Suppose you are pumping water up from a lake to a water tank. The tank is a rectangular solid, with a base that is $46'' \times 38\frac{1}{2}''$, and a height of $38''$. The base is 27 feet above the lake. Water weighs $62.4\,\mathrm{lb/ft^3}$.

   (a) How much work, in ft·lb, will it be to fill the tank?

   (b) It took about 10 oz of gasoline to pump the water up. A gallon of gasoline contains about 132 megajoules of energy, according to Wikipedia. Use the fact that 1 gallon is 128 ounces and 1 ft·lb is 1.355 joules to find the efficiency of the pump.

5. Suppose a Solo cup has radii R_1 cm and R_2 cm and height H cm.



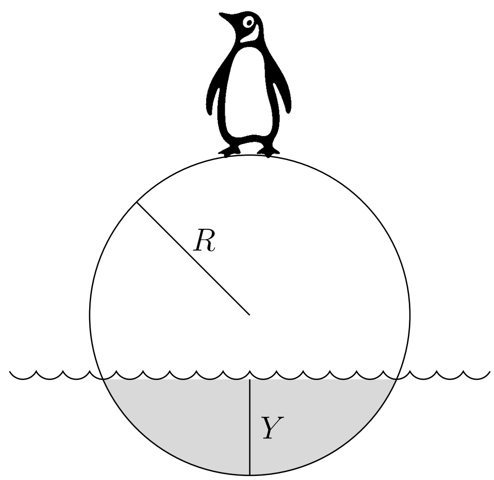
   (a) Consider a disk-shaped slice of the cup which is a height h above the bottom. What is its radius, in terms of h? Hint: The sides of the cup are straight, so the radius is a linear function of h.

   (b) If the thickness of the disk is Δh, what is its volume?

   (c) Suppose the cup is filled with water, which has density of δ kg/cm^3. How much does the slice weigh?

   (d) How much work (energy) will it take to sip all the water out of the cup?

6. The buoyancy force on a floating object is proportional to the volume of water it displaces. Using this fact, whimsical ecologists plan to study the weights of penguins by floating beachballs on the ocean and enticing penguins to climb on top. They then measure the depth the ball sinks to, and thereby deduce the penguin's weight.



   So given a beachball of radius R that is partially submerged in the water, find a formula for the volume of the ball which is below the water line when its bottom is at depth Y. Check that your formula makes sense for the values $Y = 0$, $Y = R$, and $Y = 2R$.

7. Evaluate $\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ where m and n are positive integers. (You might want to graph a few examples.)

8. Find $\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\,dx$, given that m and n are positive integers.