

Worksheet Deranged

1. Let's practice some integration by parts.

(a) $\int x^2 e^x dx$

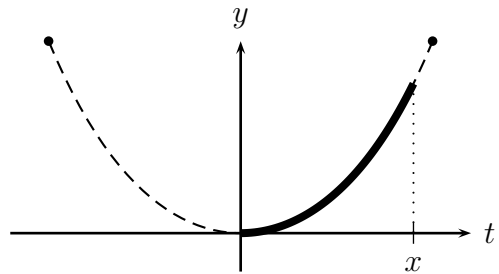
(c) $\int e^x \sin x dx$

(b) $\int \ln x dx$

(d) $\int_0^1 \tan^{-1}(x) dx$ Hint: $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$

2. We're interested in finding an equation that describes the shape of a hanging chain. Clearly the shape is determined by the forces on the chain.

(a) Consider the portion of the chain highlighted here. Draw it on the board, and draw arrows for all the forces that act on it.



(b) Give the forces names. Given that the chain is not in motion, what must the forces sum to?

(c) So how are your variables related? Write down as many equations as you can.

3. Consider the **gamma function**: $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$, for $x > 0$.

(a) Use integration by parts to prove that $\Gamma(x+1) = x\Gamma(x)$.

(b) Show that $\Gamma(1) = 1$. Then fill in this chart, using part (a):

x	1	2	3	4	5	6
$\Gamma(x)$						

(c) So if x is a positive integer, what is $\Gamma(x)$?

4. Evaluate $\int_{-\pi}^{\pi} \sin(mx) \cos(nx) dx$ where m and n are positive integers. (You might want to graph a few examples.)

5. (Fall, 2007) For this problem, $\int_1^5 g(x) dx = 12$ and $f(x) = 2x - 9$. Some values of $g(x)$ are:

x	1	2	3	4	5
$g(x)$	0.1	1.5	2	5	10

(a) Find $\int_5^7 g(f(x)) dx$.

(b) Find $\int_1^5 f(x)g'(x) dx$.

(c) Find $\int_1^5 \frac{g'(x)}{g(x)(g(x)+1)} dx$.

6. Currently 95% of Michigan kindergarteners have been vaccinated for measles. The measles vaccine is 93% effective, meaning that 7% of vaccinated children who are exposed to the disease will contract it, and the rest will not. That contrasts with a 10% immunity among unvaccinated children.

(a) Fill in the following table of possibilities. For instance, the upper-left corner is the probability that a randomly-chosen child is vaccinated *and* contracts measles.

		Vaccinated?	
		Yes	No
Gets measles?	Yes		
	No		

(b) What proportion of the students who contract measles were vaccinated?

(c) What does that mean about whether you should vaccinate your child?

7. The Michigan Lottery offers several exciting and fun ways to spend money. Let's calculate the odds of one of them.

Daily 3 Three bins, numbered 1, 2, and 3, each contain ten ping-pong balls, numbered 0 through 9. A ball is chosen from each bin, so that the result of the drawing is a 3-digit number. Players likewise choose a 3-digit number to play.

(a) What is the probability of getting all three digits correct?

(b) You can also play your numbers "boxed". That means that if you match the three digits *in any order*, you win. What is the probability of winning a boxed ticket? Does it depend on what numbers you play?