Douglass Houghton Workshop, Section 2, Tue 01/22/19

Worksheet Camel

1. Let's practice some substitution.

   (a) $\int z(z+3)^{1/3}\,dz$

   (b) $\int \dfrac{dx}{2+2\sqrt{x}}$

   (c) $\int (z+2)\sqrt{1-z}\,dz$

   (d) $\int \dfrac{3x-2}{\sqrt{2x+1}}\,dx$

2. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0. Gravity causes the ball to accelerate downward at a constant rate, g. (This might be on another planet, so use g rather than $9.8\,\mathrm{m/sec}^2$.)

   (a) Find $v(t)$, the upward velocity of the ball at time t.

   (b) Find $h(t)$, the height of the ball at time t.

   (c) Calculate the quantity $mgh(t)+\frac{1}{2}mv(t)^2$. What do you notice about your answer?

   (d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.

3. Find $\dfrac{d}{dx}\displaystyle\int_{\cos x}^{3} e^{t^2}\,dt$.

4. Find $\dfrac{d}{dx}\displaystyle\int_{g(x)}^{h(x)} f(t)\,dt$.

5. Suppose we want to compute $\displaystyle\int \frac{2x+5}{x^2-2x-3}\,dx$.

   (a) Factor the denominator into something like $(x-\alpha)(x-\beta)$.

   (b) Now reverse the process of finding a common denominator. That is, imagine the integrand can be written as

   $$\frac{A}{x-\alpha}+\frac{B}{x-\beta}$$

   for some constants A and B. Find what A and B have to be to make that the same as $\frac{2x+5}{x^2-2x-3}$.

   (c) Finally, rewrite the integral using the sum you found, and use substitution to solve it.

6. Use one of the trig identities in the front of your textbook to compute $\displaystyle\int \sin^2(x)\,dx$.

7. The Michigan Lottery offers several exciting and fun ways to spend money. Let's calculate the odds of one of them.

   Daily 3 Three bins, numbered 1, 2, and 3, each contain ten ping-pong balls, numbered 0 through 9. A ball is chosen from each bin, so that the result of the drawing is a 3-digit number. Players likewise choose a 3-digit number to play.

   (a) What is the probability of getting all three digits correct?

   (b) You can also play your numbers "boxed". That means that if you match the three digits *in any order*, you win. What is the probability of winning a boxed ticket? Does it depend on what numbers you play?

8. Define *kinetic energy* and *potential energy* of a falling object like this:

$$\text{Kinetic Energy} = \frac{1}{2}mv^2$$
$$\text{Potential Energy} = mgh$$

   where m is the object's mass, v is its velocity, and h is its height. We discovered the remarkable fact that as the object falls, the total of the two kinds of energy stays the same.

   Suppose you climb to the top of North Quad and drop an Ohio State football helmet off the top. The helmet has mass 2.1 kg, and you know that it must hit the ground with a velocity of at least 18 m/s in order to be destroyed.

   (a) How much kinetic energy must the helmet have when it hits the ground?

   (b) Find the height from which the helmet should be dropped to obtain that much energy.

   (c) How would the answers above change if the helmet were twice as heavy?

   (d) What if the desired velocity were twice as much?