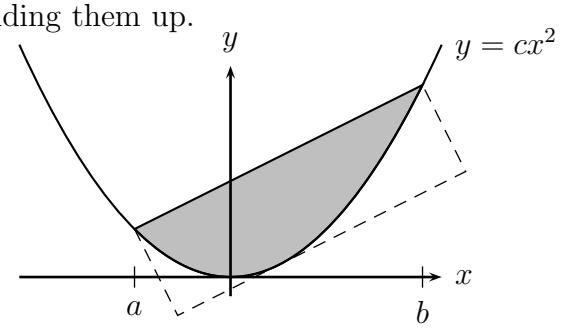
Douglass Houghton Workshop, Section 2, Tue 01/15/19

Worksheet Batrachomyomachia

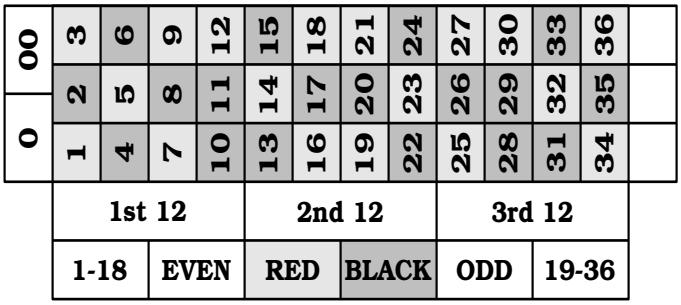
1. Find the volume of a plastic cup with radii $R_1 = 3\,\text{cm}$ and $R_2 = 4.5\,\text{cm}$, and height $H = 12\,\text{cm}$ by slicing it up into discs and adding them up.

2. (a) Find the area of the shaded region in the picture to the right. Make the answer as simple as possible.

   (b) Find the area of the dashed rectangle, which is tangent to the curve.



3. Let's calculate some probabilities for Roulette.



   (a) Suppose I put a chip on "3". That means I win if and only if the ball lands on 3. What is the probability that I win?

   (b) Fill in the table to the right with probabilities of winning the bets shown.

   (c) Suppose I keep putting a dollar chip on "red" all night long, say for 200 games. On average, how many times will I win?

   (d) The red bet pays 1:1, meaning that if I win, then I get my original dollar back, plus one more dollar. If I start the night with \$200, and play 200 times, what is the least I can have at the end? What's the most? What will I have on an average night?

Bet	Prob
1 or 2	
1 or 2 or 4 or 5	
odd	
red	
both odd and red	
either odd or red	

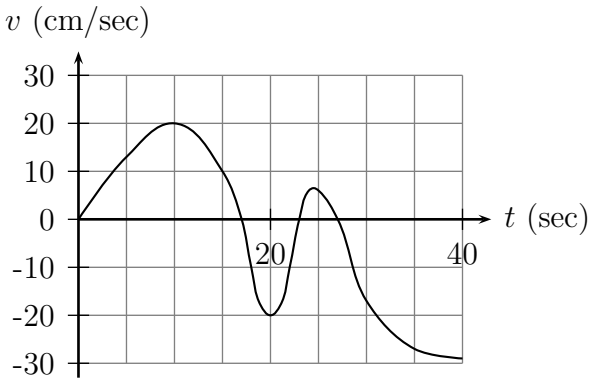
   (e) Suppose you bet on red every time the wheel spins. What's the probability you win the first two bets? How about the first 3? How about the first n?

4. Find the interval on which the graph of $f(x) = \displaystyle\int_0^x \frac{1}{1+t+t^2}\,dt$ is concave up.

5. Find $\dfrac{d}{dx}\displaystyle\int_{x^2}^{\ln(x)} e^{t^2}\,dt$.

6. (This is derived from Problem 47 on page 295 of your book.)

A mouse is trapped in a psychologist's experiment. She moves back and forth in a straight tunnel. The cruel experimenter attracts the mouse with bits of cheese at one end or the other. Sometimes he also puts a frog in the tunnel to scare the mouse away, because mice are terrified of frogs. The graph of the mouse's velocity, v, is given to the right, with a positive velocity corresponding to motion toward the right end.



Tell the story from the mouse's point of view. You might write it as a timeline, explaining what happened when. Make up explanations for all the significant features of the graph.

7. The breathing of a frog is cyclic, and when it is relaxed (because no mice are around), the time from beginning of inhalation to end of exhalation is about 5 seconds. The maximum rate of air flow into the lungs is about 50 milliliters per second.

   (a) Write a trigonometric function that models the rate of air flow into the lungs.

   (b) Use this function to find the maximum amount of inhaled air in the lungs.

8. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0. Gravity causes the ball to accelerate downward at a constant rate, g. (This might be on another planet, so use g rather than $9.8\,\mathrm{m/sec^2}$.)

   (a) Find $v(t)$, the upward velocity of the ball at time t.

   (b) Find $h(t)$, the height of the ball at time t.

   (c) Calculate the quantity $mgh(t)+\frac{1}{2}mv(t)^2$. What do you notice about your answer?

   (d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.