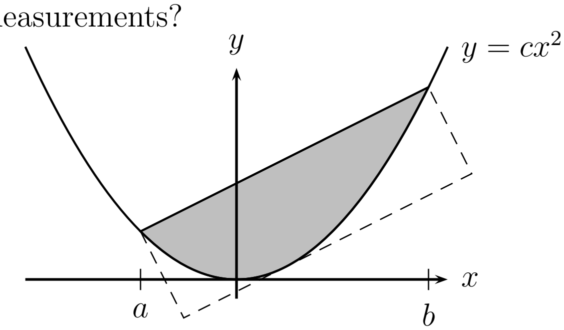
Douglass Houghton Workshop, Section 2, Thu 01/10/19

Worksheet Archimedes

1. Calculate the volume of a plastic cup, using any mathematical method you like. What assumptions do you need to make? What measurements?



2. (a) Find the area of the shaded region in the picture to the right. Make the answer as simple as possible.

   (b) Find the area of the dashed rectangle, which is tangent to the curve.

3. Let's calculate some probabilities for Roulette.

00	3	6	9	12	15	18	21	24	27	30	33	36	
	2	5	8	11	14	17	20	23	26	29	32	35	
0	1	4	7	10	13	16	19	22	25	28	31	34	
	1st 12				2nd 12				3rd 12				
	1-18		EVEN		RED		BLACK		ODD		19-36		

   (a) Suppose I put a chip on "3". That means I win if and only if the ball lands on 3. What is the probability that I win?

   (b) Fill in the table to the right with probabilities of winning the bets shown.

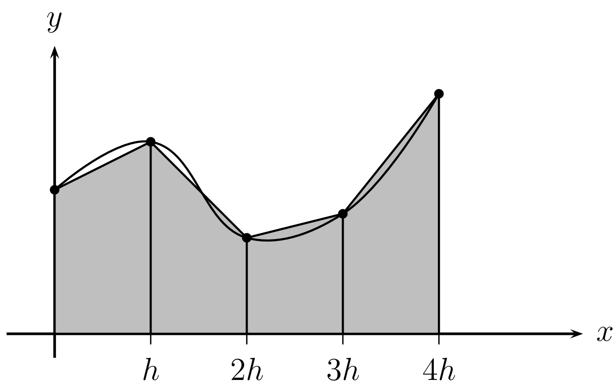
Bet	Prob
1 or 2	
1 or 2 or 4 or 5	
odd	
red	
both odd and red	
either odd or red	

   (c) Suppose I keep putting a dollar chip on "red" all night long, say for 200 games. On average, how many times will I win?

   (d) The red bet pays 1:1, meaning that if I win, then I get my original dollar back, plus one more dollar. If I start the night with $200, and play 200 times, what is the least I can have at the end? What's the most? What will I have on an average night?

   (e) Suppose you bet on red every time the wheel spins. What's the probability you win the first two bets? How about the first 3? How about the first n?

4. Last fall we talked a lot about left-hand and right-hand sums for approximating integrals. You may have thought to yourself: "Rectangles seem like a very poor approximation for the area under a curve. There's all that wasted space at the top. Can't we do better?" The truth is that rectangles are the simplest way to approximate a definite integral, but we *can* do better. The simplest improvement would be to use trapezoids instead of rectangles:



Suppose we have $f(0) = y_0, f(h) = y_1, \ldots, f(4h) = y_4$.

(a) Write a formula in terms of the y's for the left-hand sum approximation of $\int_0^{4h} f(x)\,dx$ with 4 rectangles.

(b) Write a similar formula for the right-hand sum approximation of $\int_0^{4h} f(x)\,dx$.

(c) Find the area of the leftmost trapezoid in terms of h, y_0, and y_1.

(d) Write a formula for the trapezoid approximation of $\int_0^{4h} f(x)dx$, with 4 trapezoids. How is it related to the formulas you found in (a) and (b)?

(e) How can you do better than trapezoids? Hint: First we approximated the curve with constant functions, then with line segments. What could we try next?

5. Find an example of an integral for which

(a) The left-hand, right-hand, and trapezoid estimates for $n = 4$ are all underestimates.

(b) The left-hand, right-hand, and trapezoid estimates for $n = 4$ are all overestimates.

(c) The LH and RH estimates are too high, but the trapezoid estimate is too low (again for $n = 4$).

6. Find the area of the finite region that is bounded by the y-axis, the line $y = 1$, and the graph of $y = x^{1/4}$ in two ways:

(a) By integrating with respect to x and

(b) By writing x as a function of y and integrating with respect to y.