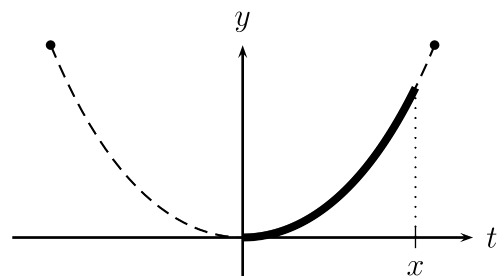
Douglass Houghton Workshop, Section 1, Mon 04/15/19

Worksheet Out of the Frying Pan...

1. We've made some progress finding the shape of a hanging chain. If the shape is given by $F(x)$, then by considering forces and arc length we've shown that

$$F''(x) = \frac{\delta g}{T_0}\sqrt{1 + F'(x)^2}$$

where T_0 is the tension at the bottom of the chain, δ is the mass density of the chain, and g is acceleration due to gravity (all constants). Where to go from here? We'd like to find a formula for $F(x)$.



   (a) Hmmm. No Fs, only F's. And lots of constants. Let $y = F'(x)$, and put all the constants together into one constant. That should make it look better.

   (b) What is y when x is 0? Now you have an initial value to go with your differential equation.

   (c) Separate the variables and Reduce the problem to an integral. DO NOT use your calculator.

   (d) Do a trig substitution to improve that integral. Hint: $\sec x = \sqrt{1 + \tan^2 x}$

2. If $f(x)$ is any differentiable function, what is the derivative of $\ln(f(x))$?

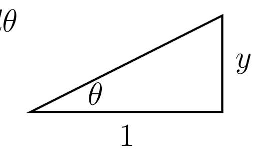
3. If $f(x) = \sec(x) + \tan(x)$, what is $f'(x)$?

4. So what is $\frac{d}{dx} \ln|\sec(x) + \tan(x)|$?

5. We are very close to finding the shape of a hanging chain. We let y be the slope of the chain, and by separating the variables of our differential equation and doing a trig substitution we found that

$$ax = \int \sec\theta \, d\theta$$

where a is some constant and $\theta = \tan^{-1} y$.



   (a) Do problems 2–4, then solve the integral to get an equation relating θ and x.

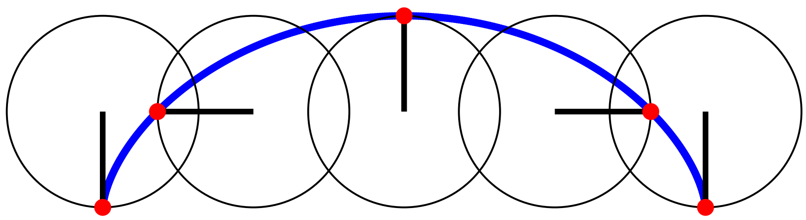
   (b) Reverse the substitution, so you have an equation relating y and x. The triangle above might be useful.

   (c) What is y when $x = 0$? Use that to resolve your constant of integration.

   (d) Solve for y in terms of x! Simplify as much as possible!

   (e) Remember that y is the *slope* of the chain, so do one more integral to get the shape of the chain.

6. Tommy is still riding a unicycle in the dark. The only thing you can see is a red reflector which is on the outer edge of the wheel, which has radius r. As Tommy moves from left to right, the reflector traces out the path below:



Progress: Previously we found a table of values for the reflector's position:

t	0 sec	1/4 sec	1/2 sec	3/4 sec	1 sec
$x_c =$ center's x	0	$\pi r/2$	πr	$3\pi r/2$	$2\pi r$
$y_c =$ center's y	0	0	0	0	0
$x_r =$ reflector's x	0	$\pi r/2 - r$	πr	$3\pi r/2 + r$	$2\pi r$
$y_r =$ reflector's y	$-r$	0	r	0	$-r$

(a) Find formulas for x_c, y_c, x_r, and y_r in terms of t. HINT on x_r: Add a line to the table for $x_r - x_c$.

(b) Find the exact distance traveled by the reflector in one minute. Solve by hand. No approximations! HINT: $\cos 2\theta = 1 - 2\sin^2\theta$.

7. (This problem appeared on the Fall, 2007 Math 116 Final Exam. Really!) Newton's law of cooling (or warming) says that the rate of change of the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. Suppose that a thermometer used by a veterinarian to find the temperature of a sick horse obeys Newton's law of cooling. Further suppose that before insertion the thermometer reads 82° F, after one minute it reads 92°, and after another minute it reads 97° F, and that a sudden convulsion unexpectedly destroys the thermometer after the 97° reading. Call the horse's temperature T_h.

(a) Write a differential equation for the temperature T (a function of time t) of the thermometer. Your equation may involve the constant T_h.

(b) Solve the differential equation for T to find a general solution for T. Your solution may include undetermined constants such as T_h.

(c) Use the temperature data to solve for T.