Douglass Houghton Workshop, Section 1, Wed 03/13/19

Worksheet Jumbo Shrimp

1. (Fall, 2014) Prove whether these series converge or diverge:

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n \ln(n)}{n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+2}}$

2. (From the Winter, 2010 Math 116 Final Exam) Discover whether the following series converge or diverge, and justify using an appropriate converge test or tests. No credit without justification.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{\sqrt{n^2+1}}{n^2-1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n!(n+1)!}{(2n)!}$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{\sin(n)}{n^2-3}$

3. Prove whether the following improper integrals converge or diverge.

   (a) $\displaystyle\int_3^{\infty} \frac{\ln(x)}{x^2}\,dx$ (b) $\displaystyle\int_0^{\infty} \frac{3}{4x^2+5\sqrt{x}}\,dx$

Any function which is continuous on $[-\pi, \pi]$ has a **Fourier series**. That is,

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

for some constants $a_0, a_1, a_2, \ldots, b_1, b_2, \ldots$ Back before spring break we discovered how to compute the constants, namely

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx, \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx, \qquad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx.$$

4. Let's compute the Fourier series for $f(x) = x^2$.

   (a) Compute a_0.

   (b) Fill in the table to the right.

   (c) Find the a_n and b_n for $f(x) = x^2$.

n	1	2	3	4	5	6
$\sin(n\pi)$						
$\sin(-n\pi)$						
$\cos(n\pi)$						
$\cos(-n\pi)$						

5. We know by the integral test that $\zeta(2) = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots + \frac{1}{n^2} + \cdots$ converges. But what does it converge to?

   (a) Use your calculator to find the first dozen or so partial sums. Can you guess what the limit is? If you like, type in the calculator program on the right and let it run, to see how the partial sums change.

   (b) Plug in $x = \pi$ to the result of the last problem and see if you can find $\zeta(2)$.

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0 → S
1 → N
Lbl 10
S+1/N² → S
N+1 → N
Disp S
Goto 10
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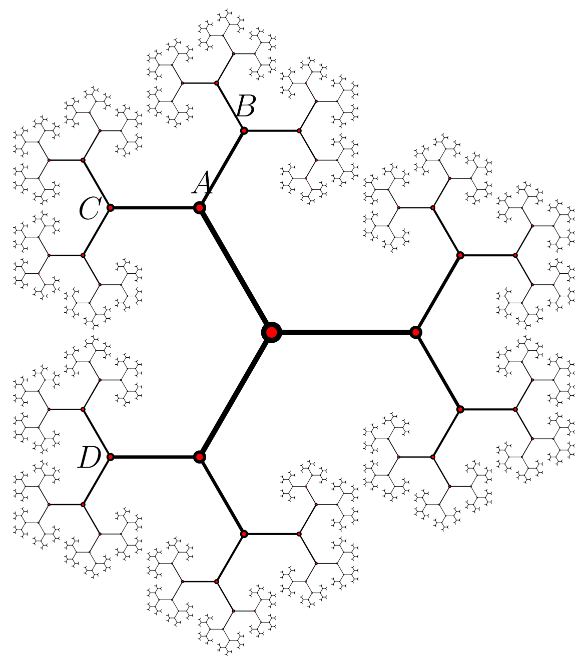
6. The figure below contains only 120° angles. As you move away from the center, the line segments get shorter by a factor r. That is, the longest segments (connected to the center) have length 1, the next longest have length r, the next longest after that have length r^2, etc. Assume the branches keep splitting and splitting, ad infinitum. Most of your answers will be in terms of r, but we'll be able to find what r is in part (h). No '...' or '$\sum$' allowed in any of your answers.



(a) Suppose you start at the center and follow the generally northward path. That is, go to A, then turn right and go to B, then turn left, right, left, etc. How far will you travel after n steps? How far will you travel if you take an infnite number of steps?

(b) If you take the path described in part (a), how far to the north will you have gone when you reach A? (That is, how much higher on the page is A than the center?) How far north will you have gone when you reach B? When you have gone n steps?

(c) Use the result of part (b) to give the total height of the figure.

(d) Find the distance from the center to the left side of the figure by following the generally northwestward path that goes to A, then turns left to C, then right, left, right, etc. This time you want the horizontal distance travelled.

(e) Find the distance from the center to the right side of the figure by following a generally northeastward path. Thus find the total width of the figure.

Now of course, the picture could be drawn with any value of r. But if r were too large, the figure would overlap itself, and if r were too small, it would hard to see what's going on. The picture above was drawn by using the largest possible value of r which doesn't cause overlap. Thus the path that goes generally southward from C never crosses the path that goes generally northward from D, but they do approach the same point.

(f) Find the vertical distance from C to D by using a path through the center.

(g) Find the same distance by considering the southward path from C and the northward path from D.

(h) Set them equal and solve for r. Do you recognize this number?