Douglass Houghton Workshop, Section 1, Wed 02/27/19

Worksheet How far that little candle throws its beams!

So shines a good deed in a naughty world.

With a lot of hard work, we filled the table to the right with the values of $\int_{-\pi}^{\pi} f\,dx(x)g(x)\,dx$, where f is the row and g is the column, and m and n are positive integers.

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$	0
$\cos(mx)$	0	0	$\begin{cases} \pi & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$

The implication was that for a function of the form

$$
\begin{aligned}
f(x) = a_0 + a_1\cos(x) + a_2\cos(2x) + a_3\cos(3x) \; &+ \; \cdots \\
+ \, b_1\sin(x) + b_2\sin(2x) + b_3\sin(3x) \; &+ \; \cdots
\end{aligned}
\tag{1}
$$

integrating against a sine or cosine function makes almost all the terms 0, so for $n \geq 1$:

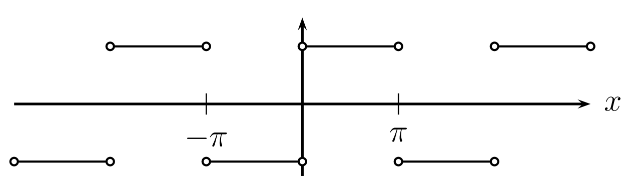
$$\int_{-\pi}^{\pi} f(x)\,dx = 2\pi a_0, \quad \int_{-\pi}^{\pi} f(x)\cos(nx)\,dx = \pi a_n, \quad \text{and} \quad \int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \pi b_n.$$

1. Generalize: Suppose that you have a function with the form of Equation (1), but you don't know the coefficients. You can, however, find integrals like the ones above.
   - How can you find a_1, the coefficient of $\cos(x)$?
   - How can you find a_n and b_n?

2. Consider the square wave:

   $$f(x) = \begin{cases} -1 & \text{if } -\pi < x < 0 \\ 1 & \text{if } \phantom{-\pi <}\ 0 < x < \pi \end{cases}$$

   and that pattern is repeated every 2π.

   Suppose the square wave can be written in terms of sines and cosines, as in Equation (1) above. Find the a_n and the b_n.

3. Find the probability of winning the pass bet in craps.

4. (This problem appeared on a Winter, 2003 Math 116 exam) Fred likes to juggle. So does Jason. The number of minutes Fred can juggle five balls without dropping one is a random variable, with probability density function $f(t) = 0.8e^{-0.8t}$. Similarly, the function $j(t) = 1.5e^{-1.5t}$ describes Jason's skill. Here t is time *in minutes*.

   (a) Find $\int_0^\infty f(t)\,dt$.

   (b) What percentage of Jason's juggling attempts are "embarrassing," meaning they last for 10 seconds or less?

   (c) How long can Fred juggle, on average?

   (d) Who is the better juggler? Give a good reason for your decision.