Douglass Houghton Workshop, Section 1, Wed 02/20/19

Worksheet Flamingo

1. Last time we found some identites for converting products of trig functions into sums:

$$\sin(x)\cos(y) = (\sin(x+y) + \sin(x-y))/2$$
$$\cos(x)\cos(y) = (\cos(x-y) + \cos(x+y))/2$$
$$\sin(x)\sin(y) = (\cos(x-y) - \cos(x+y))/2$$

Use those to finish this table:

	1	$\sin(nx)$	$\cos(nx)$
1	2π	0	0
$\sin(mx)$	0		
$\cos(mx)$	0		

with the values of $\int_{-\pi}^{\pi} f(x)g(x)\,dx$, where f is the row and g is the column.

2. Consider the "Hard Eight" bet in craps. The bet wins on double fours (⚃⚃) and loses on "soft eight" (⚁⚅ or ⚂⚄) and on 7. If something other than a 7 or 8 is rolled, the bet stays through the next roll.

   (a) Draw the addition table below on the board and fill it in.

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

   (b) Calculate these probabilities:
   - W = the probability of winning on the first roll.
   - L = the probability of losing on the first roll.
   - C = the probability that the game continues to a second roll.

   (c) Calculate the probability of winning on the *second* roll.

   (d) Calculate the probability of winning on the kth roll.

   (e) Calculate the probability of winning on *one of* the first n rolls.

   (f) Calculate the probability of winning the hard-eight bet.

3. (This problem appeared on a Winter, 2009 Math 116 exam) The quantity

$$\int_1^\infty \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

roughly models the resistance that football-shaped plankton encounter when falling through water. Note that $a = 1$, $b = 2$, and $c = 3$ are constants that describe the dimensions of the plankton. Find a value of M for which

$$\int_1^M \frac{dx}{\sqrt{(a^2+x)(b^2+x)(c^2+x)}}$$

differs from the original model of resistance by at most 0.001. Hint: make use of the integral and the comparison test.

4. A spaceship seeks to reach a height y above the surface of the earth. The force of gravity at any time is

$$\boxed{F_g = G\frac{Mm}{r^2}} \qquad \begin{aligned} G &= \text{The universal gravitational constant} \\ M &= \text{The mass of the earth} \\ m &= \text{The mass of the spaceship} \\ r &= \text{The distance from the spaceship to the center of the earth.} \end{aligned}$$

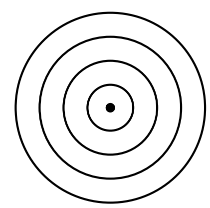
   (a) How much work will it take to raise the spaceship from the surface of the earth to a point y meters above the surface? Use R for the radius of the earth. Don't assume gravity is constant as the ship moves upward!

   (b) How much work would it take to push the spaceship entriely beyond the reach of Earth's gravity? (Let $y \to \infty$.)

   (c) If the ship is travelling at velocity v, it will have kinetic energy $\frac{1}{2}mv^2$. That energy will be converted into work to move the ship upward. What speed must the ship be going near the surface to leave the earth's gravity well? This is the earth's *escape velocity.*

   (d) Look up the values of G, M, and R, and get a numerical answer in miles per second.

5. Consider a game of "continuous darts". The board is circular, as you expect, with radius 1. The goal is to get as close to the middle as possible. If a dart lands a distance r from the bullseye, its score is $1 - r$. (So every number between 0 and 1 is a possible score.)



   A novice player throws a dart which lands randomly somewhere on the board. That means that for any region R on the board,

$$\text{Prob(dart lands in } R) = \frac{\text{area of } R}{\text{area of board}}.$$

   (a) Fill in the table with the probabilities that the dart scores **below** the given value.

x	0	1/4	1/2	3/4	1
Prob(score $< x$)					

   (b) Let x be any number. Find the probability that the score is less than x.

   (c) Find the median score.