Douglass Houghton Workshop, Section 1, Mon 01/28/19

Worksheet Damn the Torpedoes

1. Let's practice some integration by parts.

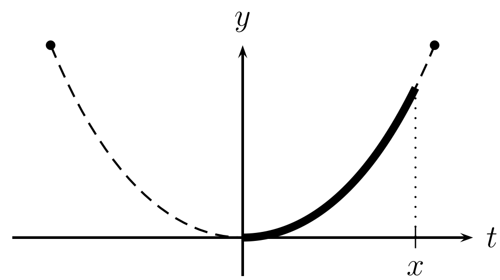
   (a) $\int x^2 e^x \, dx$

   (b) $\int \ln x \, dx$

   (c) $\int e^x \sin x \, dx$

   (d) $\int_0^1 \tan^{-1}(x) \, dx$ Hint: $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$

2. We're interested in finding an equation that describes the shape of a hanging chain. Clearly the shape is determined by the forces on the chain.

   (a) Consider the portion of the chain highlighted here. Draw it on the board, and draw arrows for all the forces that act on it.

   (b) Give the forces names. Given that the chain is not in motion, what must the forces sum to?

   (c) So how are your variables related? Write down as many equations as you can.

3. Consider the **gamma function**: $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} \, dt$, for $x > 0$.

   (a) Use integration by parts to prove that $\Gamma(x+1) = x\Gamma(x)$.

   (b) Show that $\Gamma(1) = 1$. Then fill in this chart, using part (a):

x	1	2	3	4	5	6
$\Gamma(x)$						

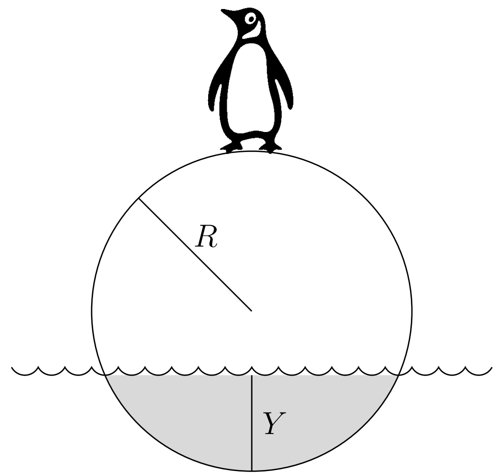
   (c) So if x is a positive integer, what is $\Gamma(x)$?

4. Suppose you are pumping water up from a lake to a water tank. The tank is a rectangular solid, with a base that is $46'' \times 38\frac{1}{2}''$, and a height of $38''$. The base is 27 feet above the lake. Water weighs $62.4\,\mathrm{lb/ft}^3$.

   (a) How much work, in ft·lb, will it be to fill the tank?

   (b) It took about 10 oz of gasoline to pump the water up. A gallon of gasoline contains about 132 megajoules of energy, according to Wikipedia. Use the fact that 1 gallon is 128 ounces and 1 ft·lb is 1.355 joules to find the efficiency of the pump.

5. The buoyancy force on a floating object is proportional to the volume of water it displaces. Using this fact, whimsical ecologists plan to study the weights of penguins by floating beachballs on the ocean and enticing penguins to climb on top. They then measure the depth the ball sinks to, and thereby deduce the penguin's weight.



So given a beachball of radius R that is partially submerged in the water, find a formula for the volume of the ball which is below the water line when its bottom is at depth Y. Check that your formula makes sense for the values $Y = 0$, $Y = R$, and $Y = 2R$.

6. Currently 95% of Michigan kindergarteners have been vaccinated for measels. The measels vaccine is 93% effective, meaning that 7% of vaccinated children who are exposed to the disease will contract it, and the rest will not. That contrasts with a 10% immunity among unvaccinated children.

   (a) Fill in the following table of possibilities. For instance, the upper-left corner is the probability that a randomly-chosen child is vaccinated *and* contracts measles.

		Vaccinated?	
		Yes	No
Gets measles?	Yes		
	No		

   (b) What proportion of the students who contract measles were vaccinated?

   (c) What does that mean about whether you should vaccinate your child?

7. Evaluate $\displaystyle\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx$ where m and n are positive integers. (You might want to graph a few examples.)