

Worksheet Casino

1. Let's practice some substitution.

(a) $\int z(z+3)^{1/3} dz$

(c) $\int (z+2)\sqrt{1-z} dz$

(b) $\int \frac{dx}{2+2\sqrt{x}}$

(d) $\int \frac{3x-2}{\sqrt{2x+1}} dx$

2. Suppose we want to compute $\int \frac{2x+5}{x^2-2x-3} dx$.

(a) Factor the denominator into something like $(x-\alpha)(x-\beta)$.

(b) Now reverse the process of finding a common denominator. That is, imagine the integrand can be written as

$$\frac{A}{x-\alpha} + \frac{B}{x-\beta}$$

for some constants A and B . Find what A and B have to be to make that the same as $\frac{2x+5}{x^2-2x-3}$.

(c) Finally, rewrite the integral using the sum you found, and use substitution to solve it.

3. Use one of the trig identities in the front of your textbook to compute $\int \sin^2(x) dx$.

4. A ball at an initial height h_0 is thrown straight up into the air, with an initial velocity v_0 . Gravity causes the ball to accelerate downward at a constant rate, g . (This might be on another planet, so use g rather than 9.8 m/sec^2 .)

(a) Find $v(t)$, the upward velocity of the ball at time t .

(b) Find $h(t)$, the height of the ball at time t .

(c) Calculate the quantity $mgh(t) + \frac{1}{2}mv(t)^2$. What do you notice about your answer?

(d) Use part (c) to calculate the maximum height of the ball. Check using your Math 115 optimization skillz.

5. Evaluate $\int_{-\pi}^{\pi} \sin(mx) \cos(nx) dx$ where m and n are positive integers. (You might want to graph a few examples.)

6. Find $\frac{d}{dx} \int_{\cos x}^3 e^{t^2} dt$.

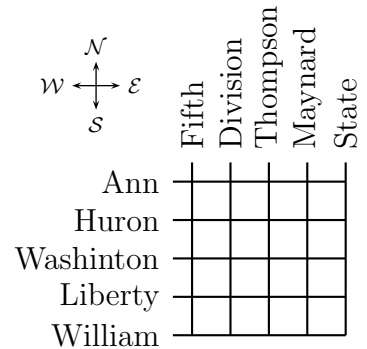
7. Find $\frac{d}{dx} \int_{g(x)}^{h(x)} f(t) dt$.

8. The Michigan Lottery offers several exciting and fun ways to spend money. Let's calculate the odds of one of them.

Daily 3 Three bins, numbered 1, 2, and 3, each contain ten ping-pong balls, numbered 0 through 9. A ball is chosen from each bin, so that the result of the drawing is a 3-digit number. Players likewise choose a 3-digit number to play.

- (a) What is the probability of getting all three digits correct?
 (b) You can also play your numbers "boxed". That means that if you match the three digits *in any order*, you win. What is the probability of winning a boxed ticket? Does it depend on what numbers you play?

9. A small section of downtown Ann Arbor is shown to the right. Copy the map onto the board.



- (a) Suppose Natalie lives at the corner of Washinton and Thompson, and she needs to get to class at Mason Hall, which is at State and William. She doesn't want to walk out of her way, so she will only go east and south. Still, she has some choices. How many ways are there to get to class?
- (b) Interesting, I wonder what that number means? Write your answer to part (a) at the corner of Washinton and Thompson. Now pick a different starting corner, and figure out how many ways there are to get to class from there. Repeat, writing your answers on the board at the relevant corner.
- (c) What's the pattern?
- (d) Explain why the pattern must continue to hold, no matter how big the city is.